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Approximating fixed points of asymptotically demicontractive mappings by iterative schemes defined as admissible perturbations

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ABSTRACT. We establish convergence theorems for a Krasnoselskij type fixed point iterative method constructed as the admissible perturbation of an asymptotically demicontractive operator defined on a convex closed subset of a Hilbert space.

1. INTRODUCTION

Let $\mathcal{H}$ be a Hilbert space and let $\mathcal{C}$ be a nonempty convex subset of $\mathcal{H}$. Let $T : \mathcal{C} \longrightarrow \mathcal{C}$ be a nonlinear mapping. Denote by $Fix(T)$ the set of fixed points of T in $\mathcal{C}$. We will assume throughout the paper that $Fix(T)$ is nonempty.

Definition 1.1 ([9]). The mapping T is called *quasi-nonexpansive* if

$$\|Tx - x^\star\| \le \|x - x^\star\|, \ \forall x \in \mathcal{C} \text{ and } x^\star \in Fix(T). \tag{1.1}$$

If the inequality (1.1) holds with strict inequality " $<$ ", then T is called *strictly quasi-nonexpansive*.

Definition 1.2 ([9]). The mapping T is called *demicontractive (k-demicontractive)* if there exists $k \in \mathbb{R}$ such that

$$\|Tx - x^\star\|^2 \le \|x - x^\star\|^2 + k\|Tx - x\|^2, \ \forall x \in \mathcal{C} \text{ and } x^\star \in Fix(T). \tag{1.2}$$

Remark 1.1. The constant k is usually considered to be in the interval $(0, 1)$. From definition we can see that for $k \ge 0$, the class of demicontractive operators includes the class of quasi-nonexpansive operators.
In the paper [1] the authors studied a class of demicontractive operators with $k < 0$, which are called strongly attractive mappings. On the other hand, in [19] the authors studied a class of k-demicontractive mappings with $k = -1$ and called them pseudo-contractive mappings.

Remark 1.2. In the particular case of $k = 1$, the k-demicontractive mappings are called hemicontractive, see for example [15] and [18].

Definition 1.3 ([9]). A mapping T with domain $\mathcal{D}(T)$ and range $\mathcal{R}(T)$ in $\mathcal{H}$ is *called strictly pseudo-contractive of Browder-Petryshyn type* [8] if, for all $x, y \in \mathcal{D}(T)$, there exists $k < 1$ such that

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + k\|x - y - (Tx - Ty)\|^2. \tag{1.3}$$

Remark 1.3. If the inequality (1.3) is true for $k = 1$ then T is called pseudocontractive, see for example [9].

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Remark 1.4. If in (1.3) we have $y \in Fix(T)$, then we obtain (1.2) and therefore T is demicontractive. It is obvious that the class of demicontractive mappings is larger than the class of strict pseudo-contractive of Browder-Petryshyn type mappings.

Definition 1.4. [13] A mapping $T : \mathcal{C} \to \mathcal{C}$ is said to be *k-strictly asymptotically pseudocontractive* ([13]) if there exists a sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = 1$ such that

$$\|T^n x - T^n y\|^2 \leq a_n^2 \|x - y\|^2 + k\| (I - T^n) x - (I - T^n) y\|^2 \tag{1.4}$$

for some $k \in [0, 1)$ and for all $x, y \in K$ and $n \in \mathbb{N}$.

T is called *asymptotically demicontractive* if $Fix\,(T) = \{x \in K : Tx = x\} \neq \emptyset$ and there exists a sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = 1$ such that

$$\|T^n x - x^*\|^2 \leq a_n^2 \|x - x^*\|^2 + k\|x - T^n x\|^2. \tag{1.5}$$

for some $k \in [0, 1)$ and for all $x \in K$, and $x^* \in F\,(T)$.

These classes of mappings were introduced by Qihou in [13] and they are larger than the classes of mappings introduced by the definitions (1.3) and (1.2).
If $k = 0$ in (1.4) then T is called *asymptotically nonexpansive*. It is clear that a k-strictly asymptotically pseudocontractive mapping with a nonempty fixed point set $Fix\,(T)$ is *asymptotically demicontractive*.

In 2012 Rus [17] introduced the theory of admissible perturbations of an operator. This theory opened a new direction of research and unified the most important aspects on the iterative approximation of fixed point for single valued self operators.

Berinde [3] continued the study of fixed point iterative methods by means of the theory of the admissible perturbations and obtained very general convergence theorems for Krasnoselskij type fixed points iterative methods defined as admissible perturbations of a nonlinear operator for the class of nonexpansive operators on Hilbert spaces.
In 2013 Ţicală presented in [23] the solvability of a class of generalized strongly nonlinear variational inequalities by using the concept of admissible perturbation operator on nonempty closed convex sets in Hilbert spaces.
In 2014 Berinde [5] obtained convergence theorems for admissible perturbations of φ - pseudocontractive operators.
In 2014 Berinde, Măruşter and Rus [6] extended the theory of admissible perturbations for nonself operators. In 2015 Ţicală [24] gave a weak convergence theorem for a Krasnoselskij type fixed point iterative method in Hilbert spaces using an admissible perturbation. In a recent paper Ţicală [25] proved some convergence theorems regarding Krasnoselskij type iterative methods defined as admissible perturbations of demicontractive operators in Hilbert spaces.

Starting from this background, in the present paper we establish some convergence theorems for a Krasnoselskij type iterative method designed to approximate fixed points of a class of asymptotically demicontractive operators. Thereby we obtain results which extend the theorems given by Qihou in [13] regarding asymptotically demicontractive operators by using admissible perturbation iterative methods and extend some of the results of Ţicală obtained in [25].

To this aim we need the results and notions presented in the next section.

2. Preliminaries

In [13], Qihou by using the modified Mann iteration method introduced by Schu [20], proved convergence theorems for the iterative approximation of fixed point of k-strictly asymptotically pseudocontractive mappings and asymptotically demicontractive mappings. We remind that a mapping T is caled uniformly L-Lipschitzian if there exists a constant $L > 0$ such that

$$\|T^n x - T^n y\| \le L\|x - y\| \tag{2.6}$$

for all $x, y \in K$ and $n \in \mathbb{N}$.

More precisely, Qihou proved the following theorems:

Theorem 2.1. [13] *Let H be a Hilbert space and K a nonempty closed convex and bounded subset of H. Let $T : K \to K$ be a completely continuous and uniformly L-Lipschitzian demicontractive mapping with sequence $\{a_n\} \subseteq [0, \infty)$, $\sum_{n=0}^{\infty} \left(a_n^2 - 1\right) < \infty$, $0 < \varepsilon \le \alpha_n \le 1 - k - \varepsilon$ for all $n \in \mathbb{N}$ and some $\varepsilon > 0$. For arbitrary $x_0 \in K$ define the sequence $\{x_n\}$ by*

$$x_{n+1} = (1 - \alpha_n) x_n + \alpha_n T^n x_n, \ n \ge 0. \tag{2.7}$$

Then $\{x_n\}$ converges strongly to some fixed point of T.

Theorem 2.2. [13] *Let H be a Hilbert space and K a nonempty closed convex and bounded subset of H. Let $T : K \to K$ be a completely continuous and uniformly L-Lipschitzian and k-strictly asymptotically pseudocontractive mapping with sequence $\{a_n\} \subseteq [1, \infty)$, $\sum_{n=0}^{\infty} \left(a_n^2 - 1\right) < \infty$, $0 < \epsilon \le \alpha_n \le 1 - k - \epsilon$, for all $n \in \mathbb{N}$ and some $\epsilon > 0$. Define the sequence $\{x_n\}$ from an arbitrary $x_0 \in K$ by*

$$x_{n+1} = (1 - \alpha_n) x_n + \alpha_n T^n x_n.$$

Then $\{x_n\}$ converges strongly to some fixed point of T.

Theorem 2.3. [26] *Let $q > 1$ and let E a real Banach space. Then E is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that*

$$\|x + y\|^q \le \|x\|^q + \langle y, j(x)\rangle + \|y\|^q \tag{2.8}$$

Lemma 2.1. *([20], p.408) Let E be a normed space and K a nonempty convex subset of E. Let $T : K \to K$ be a uniformly L-Lipschitzian mapping. For $\{\alpha_n\}$ and $\{\beta_n\}$ in $[0, 1]$, define the sequence $\{x_n\}$ from arbitrary $x_0 \in K$ by*

Lemma 2.2. [20] *Let $\{a_n\}$ and $\{b_n\}$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} < a_n + b_n, \quad n \ge 0$$

If $\sum_{n=0}^{\infty} b_n < \infty$ and $\{a_n\}$ has a subsequence which converges to 0, then $\lim_{n\to\infty} a_n = 0$

Lemma 2.3. [14] *Let $\mathcal{H}$ be a Hilbert space and K a nonempty subset of $\mathcal{H}$. Then $T : K \to K$ is a k-strictly asymptotically pseudocontractive if and only if for all $x, y \in K$*

$$Re\,\langle x - T^n x, j(x - x^*)\rangle \ge \frac{1}{2}(1 - k)\|x - T^n x\|^2 - \frac{1}{2}\left(a_n^2 - 1\right)\|x - x^*\|^2 \tag{2.9}$$

Definition 2.5. [17] *Let X be a nonempty set. A mapping $G : X \times X \to X$ is called admissible if it satisfies the following two conditions:*

$(A1)$ $G(x, x) = x$, for all $x \in X$;

$(A2)$ $G(x, y) = x$ implies $y = x$.

Definition 2.6. [17] *Let $f : X \to X$ a nonlinear operator and $G : X \times X \to X$ be an admissible mapping. The the iterative algorithm $\{x_n\}$ given by $x_0 \in X$ and*

$$x_{n+1} = G\left(x_n, f\left(x_n\right)\right),\ n \geq 0. \tag{2.10}$$

is called the Krasnoselskij algorithm corresponding to G or the GK-algorithm.

Example 2.1. [17]Let $(V, +, \mathbb{R})$ be a real vector space, $X \subset V$ a convex subset, $\lambda \in (0, 1)$, $f : X \to X$ and $G : X \times X \to X$ be defined by

$$G\left(x, f\left(x\right)\right) := \left(1 - \lambda\right) x + \lambda f\left(x\right),\ x \in X.$$

Then f_G is admissible perturbation of f, which is called the Krasnoselskij perturbation of f.

Definition 2.7. [2] Let $G : X \times X \to X$ be an admissible mapping on a normed space X. We say that G is affine Lipschitzian if there exists a constant $\mu \in [0, 1]$ such that

$$\|G\left(x_1, y_1\right) - G\left(x_2, y_2\right)\| \leq \|\mu\left(x_1 - x_2\right) + (1 - \mu)(y_1 - y_2)\| \tag{2.11}$$

for all $x_1, x_2, y_1, y_2 \in X$.

Lemma 2.4. *Let E be a normed space and K a nonempty convex subset of E. Let $T : K \to K$ be a an uniformly L-Lipschitzian mapping. For λ in $[0, 1]$ define the sequence $\{x_n\}$ from an arbitrary $x_0 \in K$ by*

$$x_{n+1} = G\left(x_n, T^n x_n\right)$$

Set $c_n = \|T^n x_n - x_n\|$, $n \geq 0$. Then $\|x_n - Tx_n\| \leq c_n + c_{n-1}\left[(1 - \lambda) + \lambda L\right]$

Proof. T is L-Lipschitzian so there exists $L > 0$ such that

$$\begin{aligned}
\|x_n - Tx_n\| &\leq \|x_n - T^n x_n\| + \|T^n x_n - Tx_n\| \\
&\leq \|x_n - T^n x_n\| + L\|T^{n-1} x_n - x_n\| \\
&\leq \|x_n - T^n x_n\| + L\|T^{n-1} x_n - T^{n-1} x_{n-1}\| + L\|T^{n-1} x_{n-1} - x_n\| \\
&\leq \|x_n - T^n x_n\| + L^2\|x_n - x_{n-1}\| + L\|T^{n-1} x_{n-1} - x_n\| \\
\|T^{n-1} x_{n-1} - x_n\| &= \|G\left(T^{n-1} x_{n-1}, T^{n-1} x_{n-1}\right) - G\left(x_{n-1}, T^{n-1} x_{n-1}\right)\| \\
&\leq \|\left(1 - \lambda\right)\left(T^{n-1} x_{n-1} - x_{x-1}\right) - \lambda\left(T^{n-1} x_{n-1} - T^{n-1} x_{n-1}\right)\| \\
&= \left(1 - \lambda\right)\|T^{n-1} x_{n-1} - x_{x-1}\|. \\
\|x_n - x_{n-1}\| &= \|G\left(x_{n-1}, T^{n-1} x_{n-1}\right) - G\left(x_{n-1}, x_{n-1}\right)\| \\
&\leq \|\left(1 - \lambda\right)\left(x_{n-1} - x_{n-1}\right) - \lambda\left(T^{n-1} x_{n-1} - x_{n-1}\right)\| \\
&= \lambda\|T^{n-1} x_{n-1} - x_{n-1}\|.
\end{aligned}$$

Finally, we have

$$\begin{aligned}
\|x_n - Tx_n\| &\leq \|x_n - T^n x_n\| + \left[L^2\lambda + L\left(1 - \lambda\right)\right]\|x_{n-1} - T^{n-1} x_{n-1}\| \\
&= c_n + c_{n-1} \cdot L \cdot \left[L^2\lambda + L\left(1 - \lambda\right)\right].
\end{aligned}$$

□

3. Main result

In this section, we obtain a result for a Krasnoselskij type iterative method defined as admissible perturbations of asymptotically demicontractive operators in Hilbert spaces. Let X, Y be Banach spaces and $T : D \subset X \longrightarrow Y$. We remind that T is said to be *completely continuous* if it is continuous and maps any bounded subset of D into a relatively compact subset of Y.

Theorem 3.4. *Let $\mathcal{H}$ be a Hilbert space. Let K be a closed convex and bounded subset of $\mathcal{H}$ and $T : K \to K$ a completely continuous uniformly L-Lipschitzian asyimptotically demicontractive mapping with sequence $\{a_n\} \subseteq [1, \infty)$ satisfying $\sum\limits_{n=0}^{\infty}\left(a_n^2 - 1\right) < \infty$.*

Let $G : K \times K \to K$ be an affine Lipschitzian admissible mapping with constant λ, which satisfies $0 < \varepsilon \leq \lambda \leq 1 - k - \varepsilon$.

Then, the sequence $\{x_n\}$ generated from an arbitrary $x_0 \in K$ by

$$x_{n+1} = G\left(x_n, T^n x_n\right) \tag{3.12}$$

converges strongly to the fixed point of T.

Proof. Let $x^* \in F(T)$. The, using the inequalities 2.8 for $q = 2$ and 2.9 we obtain

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &= \|G\left(x_n, T^n x_n\right) - G\left(x^*, x^*\right)\|^2 \\ &\leq \|\left(1 - \lambda\right)\left(x_n - x^*\right) + \lambda\left(T^n x_n - x^*\right)\|^2 \\ &= \|x_n - x^* + \lambda T^n x_n\| \\ &\leq \|x_n - x^*\|^2 - 2\lambda\left\langle x_n - T^n x_n, j\left(x_n - x^*\right)\right\rangle + \lambda^2\|T^n x_n - x_n\|^2 \end{aligned} \tag{3.13}$$

Observe that

$$\left\langle x_n - T^n x_n, j\left(x_n - x^*\right)\right\rangle \geq \frac{1}{2}\left(1 - k\right)\|x_n - T^n x_n\|^2 - \frac{1}{2}\left(a_n^2 - 1\right)\|x_n - x^*\|^2 \tag{3.14}$$

Using the inequality (3.14) in (3.13) we obtain:

$$\begin{aligned} \|x_{n+1} - x^*\| \quad &\leq \|x_n - x^*\|^2 + \lambda^2\|T^n x_n - x_n\|^2 \\ &- \left(1 - k\right)\lambda\|x_n - T^n x_n\|^2 + \left(a_n^2 - 1\right)\lambda\|x_n - x^*\| \\ &= \|x_n - x^*\|^2 + \lambda\left(a_n^2 - 1\right)\|x_n - x^*\|^2 \\ &+ \left[\lambda^2 - \lambda\left(1 - k\right)\right]\|x_n - T^n x_n\| \end{aligned} \tag{3.15}$$

Condition $0 < \varepsilon \leq \lambda \leq 1 - k - \varepsilon$ implies that

$$1 - k - \lambda \geq \varepsilon. \tag{3.16}$$

We can choose

$$\lambda \geq \frac{\varepsilon}{2}. \tag{3.17}$$

Furthermore, the boundedness of K implies that $\|x_n - x^*\|^2 \leq M$ for all $n \geq 0$and for some constant $M > 0$, so that using (3.16) and (3.17) in 3.15 we obtain

$$\|x_{n+1} - x^*\|^2 \leq \|x_n - x^*\|^2 - \frac{\varepsilon}{2} \cdot \varepsilon\|x_n - T^n x_n\|^2 + \sigma_n, \ \forall n \geq N_0. \tag{3.18}$$

where $\sigma_n = \left(a_n^2 - 1\right) \cdot \lambda \cdot M$
so that

$$\frac{\varepsilon^2}{2}\sum_{n=N_0}^{\infty}\|x_j - T^j x_j\|^2 \leq \|x_{N_0} - x^*\| + \sum_{n=N_0}^{\infty}\sigma_j \tag{3.19}$$

The condition $\sum\limits_{n=0}^{\infty}\left(a_n^2 - 1\right) < \infty$ implies that $\sum\limits_{n=0}^{\infty}\|x_n - T^n x_n\|^2 < \infty$, so that $\lim\limits_{n\to\infty}\|x_n - T^n x_n\|^2 = 0$. Thus it follows from Lemma 2.4 that

$$\lim_{n\to}\|x_n - Tx_n\| = 0. \tag{3.20}$$

Since T is completely continuous and $\{x_n\}$ is bounded, it follows that $\{Tx_n\}$ has a convergent subspace $\left\{Tx_{n_j}\right\}_{j=0}^{\infty}$, so that (3.20) implies that $\{x_n\}$ has a convergent subsequence $\left\{x_{n_j}\right\}_{j=0}^{\infty}$. Let $\lim\limits_{j\to\infty} x_{n_j} = y^*$. Then from (3.20) we obtain $y^* = Ty^*$ so that y^* is a fixed point of T. Hence, it follows from (3.18) that

$$\|x_{n+1} - y^*\|^2 \leq \|x_n - y^*\|^2 + \sigma_n,\ n \geq N_0.$$

Since $\{\|x_n - y^*\|\}$ has a subsequence which converges to 0 and $\sum\limits_{n=0}^{\infty} \sigma_n < \infty$, it follows from Lemma (2.2) that $\lim\limits_{n\to\infty} \|x_n - y^*\| = 0$, completing the proof of the Theorem. □

Remark 3.5. In order to obtain Theorem 3.4 we used as prototype a theorem in [13] where we replaced the sequence $\{a_n\}$ of the Mann iteration with the constant λ and therefore we were able to use the GK-algorithm as it was defined by Rus in [17]. In the particular case

$$G(x, y) = (1 - \lambda)\, x + \lambda y,$$

by Theorem 3.4 we obtain a convergence theorem for Krasnoselskij iteration.

This new theorem is similar to the result of Qihou [13], with the only difference that the Krasnoselskij iteration in this paper is replaced with the Mann iteration in [13].

Remark 3.6. If the mapping T is only demicontractive, then by Theorem 3.4 we obtain the convergence theorem given by Ţicală in [25].

Theorem 3.5. [25] *Let C be a closed, bounded, convex subset of a Hilbert space, $H\mathcal{H}$. Suppose T is a demicontractive mapping, $T : C \to C$ with contraction coefficient k. Suppose the set of fixed points $Fix(T)$ is nonempty.*

Suppose $G : C \times C \to C$ is an affine Lipschitzian admissible mapping with constant $\lambda < 1 - k$ then $\liminf \|x_n - Tx_n\| = 0$ *for each $x_1 \in C$ where x_{n+1} is defined by (2.10).*

4. Conclusions

We end this paper by presenting some historical notes about the classes of nonexpansive mappings.

The notion of quasi-nonexpansivity was introduced by Tricomi in 1916 [22] for a real function T defined on a finite or infinite interval (a, b) with the values in the same interval. He proved that the sequence $\{x_n\}$ generated starting from a given x_0 in (a, b) by the iteration $x_{n+1} = T(x_n)$ converges to a fixed point of T if the application T is continuous and strictly quasi-nonexpansive on (a, b). In 1975 Stepelman [21] proved that the convergence of this sequence in the real case is assured if and only if the second iterate of T is strictly quasi-nonexpansive.

The Mann iteration process is defined by the sequence $\{x_n\}$ generated from the starting point x_0 in the domain denoted by $\mathcal{D}(T)$ of the mapping T and by the iteration

$$x_{n+1} = (1 - b_n) + b_n T x_n,\ \forall x \in \mathcal{D}(T), \tag{4.21}$$

where $\{b_k\}$ is a sequence in $[0, 1]$.

The condition of demicontractivity is not sufficient for the convergence of the Mann iteration nor in infinite or finite spaces, it is necessary to impose some additional conditions on the mapping like continuity or demiclosedness [9].

Definition 4.8 ([9]). A mapping T is said to be demiclosed at y, if for any sequence $\{x_n\}$ which converges weakly to x, and if the sequence $\{Tx_n\}$ converges strongly to y, then $T(x) = y$.

Often is used the demiclosedness at 0, which is a particular case when $y = 0$.

The class of mappings satisfying the condition (1.2) and the name of demicontractive were introduced by Hick and Kubicek in 1977 [22]. They studied the convergence properties of a sequence $\{x_n\}$ generated by a Mann-type iteration to a fixed point of T in real Hilbert spaces. They proved that if T is demicontractive and $I - T$ is demiclosed at 0, then the sequence generated by the Mann-type (4.21) iteration converges weakly to a fixed point of T. The sequence $\{b_n\}$ must satisfy the condition $b_n \longrightarrow b$, $0 < t < k$, where k the demicontraction constant.

The same notion of demicontractiveness was also introduced by Măruşter in 1977 [12], independently from Hicks and Kubicek. If $\mathcal{D}(T)$ (the definition domain of T) is a closed convex subset of a Hilbert space $\mathcal{H}$, then the definition given by Măruşter is:

Definition 4.9. [12]The mapping T will be said to satisfy condition (A) if Fix(T) is nonempty and if there exists a real positive number λ such that

$$\langle x - Tx, x - \xi \rangle \geq \lambda \|x - tx\|^2,\ \forall x \in \mathcal{D}(T), \xi \in Fix(T) \tag{4.22}$$

It is routine to check that the demicontraction condition and the (A) condition are equivalent with $\lambda = (1-k)/2$. In [12] an equivalent result was proved, more specific, T satisfies the condition (A) and $I - T$ is demiclosed at zero, then the sequence x_n converges weakly to a fixed point.

The class of asymptotically nonexpansive mappings was introduced by Göebel and Kirk [10] in 1972 and reconsidered nowadays by many authors.

In 1998 Osilike [14] proved that the condition of asymptotically demicontractivity is equivalent to

$$\langle x - T^n x, x - x^\star \rangle \geq \frac{1}{2}(1-k)\|x - T^n\|^2 - \frac{1}{2}(a_n - 1)\|x - x^\star\|^2. \tag{4.23}$$

In particular, when $a_n = 1$, $\forall n$, then the condition is similar to condition (A).

It is our aim in forthcoming papers to introduce and study the convergence of some iterative methods defined as admissible perturbation in various other classes of demicontractive type mappings surveyed in [9] and references therein.

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