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Dedicated to Professor Yeol Je Cho on the occasion of his retirement

Levitin-Polyak well-posedness for parametric quasivariational inclusion and disclusion problems

PANATDA BOONMAN and RABIAN WANGKEEREE

ABSTRACT. In this paper, we aim to suggest the new concept of Levitin-Polyak (for short, LP) well-posedness for the parametric quasivariational inclusion and disclusion problems (for short, (QVIP) (resp. (QVDP))). Necessary and sufficient conditions for LP well-posedness of these problems are proved. As applications, we obtained immediately some results of LP well-posedness for the quasiequilibrium problems and for a scalar equilibrium problem.

1. INTRODUCTION

Well-posedness is very important concept in optimization theory, for well-posed optimization problems, which guarantees that, for every approximating solution sequence, there is a subsequence which converges to a solution. In 1966, well-posedness of unconstrained and constrained scalar optimization problems was first introduced and studied by Tykhonov [24] and Levitin and Polyak [15], respectively. Well-posedness for various problems related to optimization has been recently intensively considered, see e.g: for optimization problems [11, 12, 13, 21, 23, 31, 32], for variational inequalities [5, 7, 9, 10, 17, 25], for Nash equilibria [18, 20], for inclusion problems [10, 26, 27, 28], for equilibrium problems [2, 8, 16, 30] and for fixed point problems [6, 10, 22].

Lin and Chuang [19] studied and extended the well-posedness to variational inclusion and disclusion problems and optimization problems with variational inclusion and disclusion problems as constraints. They proved some results concerned with the well-posedness in the generalized sense, the well-posedness for optimization problems for variational inclusion problems and variational disclusion problems and scalar equilibrium problems as constraint. Recently, Wang and Huang [26] introduced and studied LP well-posedness for generalized quasivariational inclusion and disclusion problems. Necessary and sufficient conditions for LP well-posedness of these problems are proved.

On the other hand, in [3], Anh, Khanh and Quy introduced and studied the parametric generalized quasivariational inclusion problem (QVIP) which contains many kinds of problems such as generalized quasivariational inclusion problems, quasioptimization problems, quasiequilibrium problems, quasivariational inequalities, complementarity problems, vector minimization problems, Nash equilibria, fixed-point and coincidence-point problems, traffic networks, etc. It is well known that a quasioptimization problem is more general than an optimization one as constraint sets depend on the decision variable as well. It is investigated in [3] the semicontinuity properties of solution maps to (QVIP). In 2016, Wangkeeree, Anh and Boonman [29] studied the new concept of well-posdness for

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Corresponding author: Rabian Wangkeeree; rabianw@nu.ac.th

the general parametric quasivariational inclusion problems (QVIP). The corresponding concepts of well-poseness in the generalized sense are also introduced and investigated for (QVIP). Some metric characterizations of well-posedness for (QVIP) are also studied.

Motivated and inspired by the works mentioned above [3, 19, 26, 29], there is no work to provide the concept of LP well-posedness for (QVIP) (resp. (QVDP)). In this paper, our main aim is to suggest the new concept of LP well-posedness for (QVIP) (resp. (QVDP)). Necessary and sufficient conditions for LP well-posedness of these problems are proved. As applications, we obtained immediately some results of LP well-posedness for the quasiequilibrium problems and for a scalar equilibrium problem.

2. Preliminaries

Let X and Y be two metric spaces, $T : X \to 2^Y$ be a multivalued map. T is said to be upper semicontinuous (u.s.c., shortly) (resp. lower semicontinuous (l.s.c., shortly)) at $x_0 \in X$ if for any open set $V \subseteq Y$, where $T(x_0) \subseteq V$ (resp. $T(x_0) \cap V \neq \emptyset$), there exists a neighborhood $U \subseteq X$ of x_0 such that $T(x) \subseteq V$ (resp. $T(x) \cap V \neq \emptyset$), $\forall x \in U$; $T(\cdot)$ is said to be u.s.c. (resp. l.s.c.) on X if it is u.s.c. (resp. l.s.c.) at every $x \in X$; T is continuous on X if it is both u.s.c. and l.s.c. on X; T is closed if $\mathrm{gr}(T) := \{(x, y) \in X \times Y \mid y \in T(x)\}$ is a closed set $X \times Y$; T is open if graph of T is open in $X \times Y$.

Lemma 2.1. [4] *Let X and Y be two metric spaces, $T : X \to 2^Y$ a multivalued mapping.*

(i) If T is u.s.c. and closed-valued, then T is closed.

(ii) If T is u.s.c. at $\bar{x}$ and $T(\bar{x})$ is compact, then for any sequence $\{x_n\}$ converging to $\bar{x}$, every sequence $\{y_n\}$ with $y_n \in T(x_n)$ has a subsequence convering to some point in $T(\bar{x})$. If, in addition, $T(\bar{x}) = \{\bar{y}\}$ is a singleton, then such a sequnece $\{y_n\}$ must converge to $\bar{y}$.

(iii) T is l.s.c. at $\bar{x}$ if and only if for any sequence $\{x_n\}$ with $x_n \to \bar{x}$ and any point $y \in T(\bar{x})$, there is a sequence $\{y_n\}$ with $y_n \in S(x_n)$ converging to y.

Definition 2.1. [14] Let (E, d) be a complete metric space. The Kuratowski measure of noncompactness of subset M of E is defined by

$$\mu(M) = \inf\left\{\varepsilon > 0 : M \subseteq \bigcup_{i=1}^{n} M_i \text{ and } \mathrm{diam} M_i < \varepsilon, i = 1, 2, \ldots, n\right\},$$

where $\mathrm{diam} M_i$ denotes the diameter of M_i and is defined by $\mathrm{diam} M_i = \sup\{d(x_1, x_2) : x_1, x_2 \in M_i\}$.

Definition 2.2. Let A and B be nonempty subset of a metric space (E, d). The Hausdorff distance $\mathcal{H}(\cdot, \cdot)$ between A and B is defined by $\mathcal{H}(A, B) := \max\{H^*(A, B), H^*(B, A)\}$, where $H^*(A, B) := \sup_{a\in A} d(a, B)$ with $d(a, B) = \inf_{b\in B} d(a, b)$.

Lemma 2.2. [14] *Let (X, d) be a complete metric space. If (F_n) is a decreasing sequence of nonempty, closed and bounded subsets of X such that $\lim_{n\to\infty} \mu(F_n) = 0$, then the intersection $F_\infty = \bigcap_{n=1}^{\infty} F_n$ is a nonempty and compact subset of X.*

3. LP well-posedness for parametric quasivariational inclusion and disclusion problems

Throughout this article, unless otherwise specified, we use the following notations. Let (E, d) and (E', d') be two metric spaces and X and Λ be nonempty closed subsets of E and E', respectively. Let Z be a Hausdorff topological vector space. Let $K_1, K_2 : X \times \Lambda \to 2^X$ and $F_1, F_2 : X \times X \times \Lambda \to 2^Z$ be multivalued mappings. Let $e : X \to Z$ be a continuous mapping. We consider the following *parametric quasivariational inclusion and disclusion problems*, for each $\lambda \in \Lambda$,

$(\text{QVIP})_\lambda$: Finding $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $0 \in F_1(\bar{x}, y, \lambda)$, for each $y \in K_2(\bar{x}, \lambda)$;
$(\text{QVDP})_\lambda$: Finding $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $0 \notin F_2(\bar{x}, y, \lambda)$, for each $y \in K_2(\bar{x}, \lambda)$.

Denote by (QVIP) (resp. (QVDP)) the families $\{(\text{QVIP})_\lambda : \lambda \in \Lambda\}$ (resp. $\{(\text{QVDP})_\lambda : \lambda \in \Lambda\}$). For each $\lambda \in \Lambda$, let $S_{(\text{QVIP})_\lambda}$ (resp. $S_{(\text{QVDP})_\lambda}$) be solution sets of $(\text{QVIP})_\lambda$ (resp.$(\text{QVDP})_\lambda$).

For each $a \in E$ and each $r > 0$, we denote by $B(a, r)$ the closed ball centered at a with radius r. When $E = \mathbb{R}$, we denote by $B^+(0, r)$ the closed interval $[0, r]$.

Definition 3.3. Let $\lambda \in \Lambda$ and let $\{\lambda_n\} \subseteq \Lambda$ be any sequence such that $\lambda_n \to \lambda$. A sequence $\{x_n\} \subseteq X$ is called a *LP approximating solution sequence* for $(\text{QVIP})_\lambda$ if there exists a sequence $\{\varepsilon_n\}$ of positive real numbers with $\varepsilon_n \to 0$ such that, for each $n \in \mathbb{N}$, $d(x_n, K_1(x_n, \lambda_n)) \leq \varepsilon_n$ and $0 \in F_1(x_n, y, \lambda_n) + B^+(0, \varepsilon_n)e(x_n), \ \forall y \in K_2(x_n, \lambda_n)$.

Definition 3.4. Let $\lambda \in \Lambda$ and let $\{\lambda_n\} \subseteq \Lambda$ be any sequence such that $\lambda_n \to \lambda$. A sequence $\{x_n\} \subseteq X$ is called a *LP approximating solution sequence* for $(\text{QVDP})_\lambda$ if there exists a sequence $\{\varepsilon_n\}$ of positive real numbers with $\varepsilon_n \to 0$ such that, for each $n \in \mathbb{N}$, $d(x_n, K_1(x_n, \lambda_n)) \leq \varepsilon_n$ and $0 \notin F_2(x_n, y, \lambda_n) + B^+(0, \varepsilon_n)e(x_n), \ \forall y \in K_2(x_n, \lambda_n)$.

Definition 3.5. (i) (QVIP) is said to be *LP well-posed* if for every $\lambda \in \Lambda$, $(\text{QVIP})_\lambda$ has a unique solution x_λ, and for every sequence $\{\lambda_n\} \subseteq \Lambda$ with $\lambda_n \to \lambda$, every approximating solution sequence for $(\text{QVIP})_\lambda$ corresponding to $\{\lambda_n\}$ converges to x_λ, and (QVIP) is said to be *LP well-posed in the generalized sense* if for every $\lambda \in \Lambda$, $(\text{QVIP})_\lambda$ has a nonempty solution set $S_{(\text{QVIP})_\lambda}$, and for every sequence $\{\lambda_n\} \subseteq \Lambda$ with $\lambda_n \to \lambda$, every approximating solution sequence for $(\text{QVIP})_\lambda$ corresponding to $\{\lambda_n\}$ has a subsequence which converges to a point of $S_{(\text{QVIP})_\lambda}$.

(ii) (QVDP) is said to be *LP well-posed* if for every $\lambda \in \Lambda$, $(\text{QVDP})_\lambda$ has a unique solution x_λ, and for every sequence $\{\lambda_n\} \subseteq \Lambda$ with $\lambda_n \to \lambda$, every approximating solution sequence for $(\text{QVDP})_\lambda$ corresponding to $\{\lambda_n\}$ converges to x_λ, and (QVDP) is said to be *LP well-posed in the generalized sense* if for every $\lambda \in \Lambda$, $(\text{QVDP})_\lambda$ has a nonempty solution set $S_{(\text{QVDP})_\lambda}$, and for every sequence $\{\lambda_n\} \subseteq \Lambda$ with $\lambda_n \to \lambda$, every approximating solution sequence for $(\text{QVDP})_\lambda$ corresponding to $\{\lambda_n\}$ has a subsequence which converges to a point of $S_{(\text{QVDP})_\lambda}$.

Remark 3.1. Definition 3.3 generalizes Definition 3.1 of [29]. Indeed, the condition (i) of Definition 3.1 in [29] "$x_n \in K_1(x_n, \lambda_n)$ ", implies that $d(x_n, K_1(x_n, \lambda_n)) = 0$. So, Definition 3.3 generalizes Definition 3.1 of [29].

For each $\lambda \in \Lambda$, the approximating solution set for $(\text{QVIP})_\lambda$ and $(\text{QVDP})_\lambda$, respectively, are defined by, for all $\delta, \varepsilon > 0$, $\Omega_{(\text{QVIP})_\lambda}(\delta, \varepsilon) = \bigcup_{\lambda' \in B(\lambda, \delta)} \widetilde{S}_{(\text{QVIP})_\lambda}(\lambda', \varepsilon)$,
where $\widetilde{S}_{(\text{QVIP})_\lambda} : \Lambda \times \mathbb{R}^+$ is defined by, for all $\lambda' \in \Lambda, \varepsilon \in \mathbb{R}^+$,

$$\widetilde{S}_{(\text{QVIP})_\lambda}(\lambda', \varepsilon) := \left\{ x \in X \;\middle|\; \begin{array}{l} d(x, K_1(x, \lambda')) \leq \varepsilon \text{ and} \\ 0 \in F_1(x, y, \lambda') + B^+(0, \varepsilon)e(x), \forall y \in K_2(x, \lambda') \end{array} \right\}, \tag{3.1}$$

and $\Omega_{(\text{QVDP})_\lambda}(\delta, \varepsilon) = \bigcup_{\lambda' \in B(\lambda, \delta)} \widetilde{S}_{(\text{QVDP})_\lambda}(\lambda', \varepsilon)$, where $\widetilde{S}_{(\text{QVDP})_\lambda} : \Lambda \times \mathbb{R}^+$ is defined by, for all $\lambda' \in \Lambda, \varepsilon \in \mathbb{R}^+$,

$$\widetilde{S}_{(\text{QVDP})_\lambda}(\lambda', \varepsilon) := \left\{ x \in X \;\middle|\; \begin{array}{l} d(x, K_1(x, \lambda')) \leq \varepsilon \text{ and} \\ 0 \notin F_2(x, y, \lambda') + B^+(0, \varepsilon)e(x), \forall y \in K_2(x, \lambda') \end{array} \right\}. \tag{3.2}$$

Clearly, we have, for every $\lambda \in \Lambda$, (i) $S_{(\text{QVIP})_\lambda} \equiv \widetilde{S}_{(\text{QVIP})_\lambda}(\lambda, 0) \subseteq \Omega_{(\text{QVIP})_\lambda}(\delta, \varepsilon), \ \forall \delta, \varepsilon > 0$ and $S_{(\text{QVDP})_\lambda} \equiv \widetilde{S}_{(\text{QVDP})_\lambda}(\lambda, 0) \subseteq \Omega_{(\text{QVDP})_\lambda}(\delta, \varepsilon), \ \forall \delta, \varepsilon > 0$, (ii) if $0 < \delta_1 \leq \delta_2$ and $0 < \varepsilon_1 \leq \varepsilon_2$, then $\Omega_{(\text{QVIP})_\lambda}(\delta_1, \varepsilon_1) \subseteq \Omega_{(\text{QVIP})_\lambda}(\delta_2, \varepsilon_2)$ and $\Omega_{(\text{QVDP})_\lambda}(\delta_2, \varepsilon_2) \subseteq \Omega_{(\text{QVDP})_\lambda}(\delta_1, \varepsilon_1)$.

Lemma 3.3. *Assume that K_1 is closed-valued and u.s.c. and K_2 is l.s.c..*

(i) *If, for each* $x \in X$, $F_1(x,.,.)$ *is closed, then* $S_{(\mathrm{QVIP})_\lambda} = \bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon)$ *for each* $\lambda \in \Lambda$.

(ii) *If, for each* $x \in X$, $F_2(x,.,.)$ *is open, then* $S_{(\mathrm{QVDP})_\lambda} = \bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVDP})_\lambda}(\delta,\varepsilon)$ *for each* $\lambda \in \Lambda$.

Proof. (i) For any given $\lambda \in \Lambda$, it is clear that $S_{(\mathrm{QVIP})_\lambda} \subseteq \bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon)$. Thus, we only need to show that $\bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon) \subseteq S_{(\mathrm{QVIP})_\lambda}$. Suppose on the contrary that there exists $x^* \in \bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon)$ such that $x^* \notin S_{(\mathrm{QVIP})_\lambda}$. Then, for each $\delta > 0$ and each $\varepsilon > 0$, $x^* \in \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon)\setminus S_{(\mathrm{QVIP})_\lambda}$. In particular, for each $n \in \mathbb{N}$, we have $x^* \in \Omega_{(\mathrm{QVIP})_\lambda}\left(\frac{1}{n},\frac{1}{n}\right)\setminus S_{(\mathrm{QVIP})_\lambda}$, and so there exists $\lambda_n \in B(\lambda,\frac{1}{n})$ such that

$$d(x^*, K_1(x^*,\lambda_n)) \leq \frac{1}{n}, \text{ and} \tag{3.3}$$

$$0 \in F_1(x^*,y,\lambda_n) + B^+\Big(0,\frac{1}{n}\Big)e(x^*), \;\; \forall y \in K_2(x^*,\lambda_n). \tag{3.4}$$

Obviously, $\lambda_n \to \lambda$. Since K_1 is closed-valued, it follow from (3.3) that we can choose $x_n \in K_1(x^*,\lambda_n)$ such that $d(x^*,x_n) \leq \frac{1}{n}$, $\forall n \in \mathbb{N}$. Thus, $x_n \to x^*$ as $n \to \infty$. Since K_1 is closed-valued and u.s.c., we have K_1 is closed, it follows that $x^* \in K_1(x^*,\lambda)$. We observe that for each $y \in K_2(x^*,\lambda)$, since K_2 is l.s.c. at (x^*,λ) and $(x^*,\lambda_n) \to (x^*,\lambda)$, there exists $y_n \in K_2(x^*,\lambda_n)$ such that $y_n \to y$. Applying (3.4), we have that $0 \in F_1(x^*,y_n,\lambda_n) + B^+\Big(0,\frac{1}{n}\Big)e(x^*)$. Thus, there exists a sequence $\{\gamma_n\} \subseteq B^+(0,\frac{1}{n})$ such that, for each $n \in \mathbb{N}$, $0 \in F_1(x^*,y_n,\lambda_n) + \gamma_n e(x^*)$, which gives that $-\gamma_n e(x^*) \in F_1(x^*,y_n,\lambda_n)$ that is $((y_n,\lambda_n),-\gamma_n e(x^*)) \in \mathrm{Gr}(F_1(x^*,.,.))$. It is clear that $\{((x^*,y_n,\lambda_n),-\gamma_n e(x^*))\} \to ((x^*,y,\lambda),0)$. The closedness of the mapping $F_1(x,.,.)$ implies that $((y,\lambda),0) \in \mathrm{Gr}(F_1(x^*,.,.))$. That is $0 \in F_1(x^*,y,\lambda)$ and so $x^* \in S_{(\mathrm{QVIP})_\lambda}$, which is a contradiction. Hence $\bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon) \subseteq S_{(\mathrm{QVIP})_\lambda}$. (ii) For any given $\lambda \in \Lambda$ and let $F_1 : X \times X \times \Lambda \to 2^Z$ be defined by $F_1(x,y,\lambda) = Z\setminus F_2(x,y,\lambda)$ for each $(x,y,\lambda) \in X \times X \times \Lambda$. Then $S_{(\mathrm{QVIP})_\lambda} = S_{(\mathrm{QVDP})_\lambda}$. For each $\delta > 0$ and $\varepsilon > 0$ we have $\Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon) = \Omega_{(\mathrm{QVDP})_\lambda}(\delta,\varepsilon)$. Since $F_2(x,.,.)$ is open, we have $F_1(x,y,\lambda)$ is closed. By (i), the proof is complete. □

The following example is given to illustrate the case that Lemma 3.3 is applicable.

Example 3.1. Let $E = Z = \mathbb{R}$, $X = [0,+\infty)$ and $\Lambda = [0,1]$. For every $(x,y,\lambda) \in X \times X \times \Lambda$, let $e(x) = x^2$, $K_1(x,\lambda) = [\lambda^2,+\infty)$ and $K_2(x,\lambda) = [x^2+\lambda^2, x^2+1]$. Define a set-valued mapping $F_1, F_2 : X \times X \times \Lambda \to 2^Z$ by $F_1(x,y,\lambda) = (-\infty, 2x-y+\lambda]$, $F_2(x,y,\lambda) = (2x-y+\lambda,+\infty)$. Obviously, it is to see that all assumptions of Lemma 3.3 are satisfied. Hence, $S_{(\mathrm{QVIP})_\lambda} = \bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon)$ and $S_{(\mathrm{QVDP})_\lambda} = \bigcap_{\delta>0,\varepsilon>0} \Omega_{(\mathrm{QVDP})_\lambda}(\delta,\varepsilon)$ for each $\lambda \in \Lambda$.

Lemma 3.4. *For* (QVIP) *and* (QVDP), assume that K_1 is closed-valued and u.s.c. and K_2 is l.s.c..

(i) If, for each $\lambda \in \Lambda$, $F_1(.,.,\lambda)$ is closed and K_1 is also compact-valued, then for each $(\lambda,\varepsilon) \in \Lambda \times \mathbb{R}^+$, $\widetilde{S}_{(\mathrm{QVIP})_\lambda}(\lambda,\varepsilon)$ is closed subset of X, where $\widetilde{S}_{(\mathrm{QVIP})_\lambda}$ is defined by (3.1) and so is $\Omega_{(\mathrm{QVIP})_\lambda}(\delta,\varepsilon)$.

(ii) If, for each $\lambda \in \Lambda$, $F_2(.,.,\lambda)$ is open and K_1 is also compact-valued, then for each $(\lambda,\varepsilon) \in \Lambda \times \mathbb{R}^+$, $\widetilde{S}_{(\mathrm{QVDP})_\lambda}(\lambda,\varepsilon)$ is closed subset of X, where $\widetilde{S}_{(\mathrm{QVDP})_\lambda}$ is defined by (3.2) and so is $\Omega_{(\mathrm{QVDP})_\lambda}(\delta,\varepsilon)$.

Proof. Let $(\lambda, \varepsilon) \in \Lambda \times \mathbb{R}^+$ be fixed and suppose that K_1 is also compact-valued. If $x \in \mathrm{cl}\widetilde{S}_{(\mathrm{QVIP})_\lambda}(\lambda, \varepsilon)$, then there exists a sequence $\{x_n\} \subseteq \widetilde{S}_{(\mathrm{QVIP})_\lambda}(\lambda, \varepsilon)$ such that $x_n \to x$ as $n \to \infty$. It follows that, for each $n \in \mathbb{N}$, $x_n \in X$ such that for each $y \in K_2(x_n, \lambda)$,

$$d(x_n, K_1(x_n, \lambda)) \leq \varepsilon, \text{ and} \tag{3.5}$$

$$0 \in F_1(x_n, y, \lambda) + B^+(0, \varepsilon)e(x_n). \tag{3.6}$$

By (3.5), for each $n \in \mathbb{N}$, there exists $u_n \in K_1(x_n, \lambda)$ such that

$$d(x_n, u_n) \leq \varepsilon + \frac{1}{n}. \tag{3.7}$$

Since K_1 is u.s.c. and compact-valued, there exists a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ such that $u_{n_k} \to u$ as $k \to \infty$. It follows that $d(x, u) = \lim_{k\to\infty} d(x_{n_k}, u_{n_k}) \leq \varepsilon$. Since K_1 is u.s.c. and closed-valued, we have K_1 is closed. Thus $u \in K_1(x, \lambda)$. This implies that

$$d(x, K_1(x, \lambda)) \leq \varepsilon. \tag{3.8}$$

For each $y \in K_2(x, \lambda)$, since K_2 is l.s.c., there exists a sequence $\{y_n\}$ with $y_n \in K_2(x_n, \lambda)$ such that $y_n \to y$ as $n \to \infty$. By (3.6), we have $0 \in F_1(x_n, y_n, \lambda) + B^+(0, \varepsilon)e(x_n), \ \forall n \in \mathbb{N}$. Thus there exists a sequence $\{\alpha_n\} \subseteq B^+(0, \varepsilon)$ such that $0 \in F_1(x_n, y_n, \lambda) + \alpha_n e(x_n), \ \forall n \in \mathbb{N}$. Observe that $B^+(0, \varepsilon) := [0, \varepsilon] \subseteq \mathbb{R}$ is compact. Assume that $\alpha_n \to \alpha \in B^+(0, \varepsilon)$ as $n \to \infty$. Since $F_1(.,.,\lambda)$ is closed, one has $0 \in F_1(x, y, \lambda) + \alpha e(x) \subseteq F_1(x, y, \lambda) + B^+(0, \varepsilon)e(x)$. Therefore $x \in \widetilde{S}_{(\mathrm{QVIP})_\lambda}(\lambda, \varepsilon)$, and this implies that $\widetilde{S}_{(\mathrm{QVIP})_\lambda}(\lambda, \varepsilon)$ is a closed subset of X. Now it follows $\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)$ is a closed subset of X. (ii) Let $F_1 : X \times X \times \Lambda \to 2^Z$ be defined by $F_1(x, y, \lambda) = Z \backslash F_2(x, y, \lambda)$ for each $(x, y, \lambda) \in X \times X \times \Lambda$. Then $\widetilde{S}_{(\mathrm{QVIP})_\lambda}(\lambda, \varepsilon) = \widetilde{S}_{(\mathrm{QVDP})_\lambda}(\lambda, \varepsilon)$ and $S_{(\mathrm{QVIP})_\lambda} = S_{(\mathrm{QVDP})_\lambda}$, and so $\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon) = \Omega_{(\mathrm{QVDP})_\lambda}(\delta, \varepsilon)$. Since $F_2(.,.,\lambda)$ is open, we have $F_1(x, y, \lambda)$ is closed. By (i), the proof is complete. □

If E is finite-dimension normed space, then the assumption that "K_1 is also compact-valued in Lemma 3.4 " can be removed

Lemma 3.5. *Let E be finite-dimensional normed space. For (QVIP) and (QVDP), assume that K_1 is closed-valued and u.s.c. and K_2 is l.s.c..*

- *(i) If, for each $\lambda \in \Lambda$, $F_1(.,.,\lambda)$ is closed, then $S_{(QVIP)_\lambda}$, $\widetilde{S}_{(QVIP)_\lambda}(\lambda, \varepsilon)$ and $\Omega_{(QVIP)_\lambda}(\delta, \varepsilon)$ are closed subset of X.*
- *(ii) If, for each $\lambda \in \Lambda$, $F_2(.,.,\lambda)$ is open, then $S_{(QVDP)_\lambda}$, $\widetilde{S}_{(QVDP)_\lambda}(\lambda, \varepsilon)$ and $\Omega_{(QVDP)_\lambda}(\delta, \varepsilon)$ are closed subset of X.*

Proof. We can proceed the proof exactly as that of Lemma 3.4 except for using the Assumption that E is finite-dimension normed space to get $d(x, K_1(x, \lambda)) \leq \varepsilon$. In fact, since $x_n \to x$, it follows that $\{x_n\}$ is bounded. By (3.7), we have $\{u_n\}$ is also bounded. Thus there exists a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ such that $\{u_{n_k}\}$ converges to some $u \in X$ as $k \to \infty$. Since K_1 is closed-valued and u.s.c., we have K_1 is closed, it follows that $u \in K_1(x, \lambda)$. It follows that $d(x, u) = \lim_{k\to\infty} d(x_{n_k}, u_{n_k}) \leq \varepsilon$ and so $d(x, K_1(x, \lambda)) \leq \varepsilon$. This complete the proof. □

Remark 3.2. If $K_1(x, \lambda) \equiv K_2(x, \lambda) \equiv X$, then our problem (QVIP) reduces to (VIP) in Lin and Chuang [19].

Now, we are in a position to state and prove our main results.

Theorem 3.1. *For* (QVIP)*, assume that E is complete, K_1 is closed-valued and u.s.c., K_2 is l.s.c. and F_1 is closed. Then* (QVIP) *is LP well-posed if and only if for every $\lambda \in \Lambda$,*

$$\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon) \neq \emptyset, \ \forall \delta, \varepsilon > 0, \ \textit{and} \ \operatorname{diam}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)) \to 0 \textit{ as } (\delta, \varepsilon) \to (0, 0). \tag{3.9}$$

Proof. Supposed that (QVIP) is LP well-posed. Then, for every $\lambda \in \Lambda, (\text{QVIP})_\lambda$ has a unique solution x_λ, $S_{(\text{QVIP})_\lambda} \neq \emptyset$, and so $\Omega_\lambda(\delta,\varepsilon) \neq \emptyset$, for all $\delta, \varepsilon > 0$. Now we shall show that

$$\text{diam}(\Omega_{(\text{QVIP})_\lambda}(\delta,\varepsilon)) \to 0 \text{ as } (\delta,\varepsilon) \to (0,0). \tag{3.10}$$

Suppose to the contrary the existences of (3.10), there exist $l > 0$, sequences $\{\delta_n\}$ and $\{\varepsilon_n\}$ of positive real numbers with $(\delta_n,\varepsilon_n) \to (0,0)$ as $n \to \infty$ and sequence $\{x_n\}$ and $\{x'_n\}$ with $x_n, x'_n \in \Omega_{(\text{QVIP})_\lambda}(\delta_n,\varepsilon_n)$ for each $n \in \mathbb{N}$ such that

$$d(x_n, x'_n) > l, \ \ \forall n \in \mathbb{N}. \tag{3.11}$$

For each $n \in \mathbb{N}$, since $x_n \in \Omega_{(\text{QVIP})_\lambda}(\delta_n,\varepsilon_n)$, there exists $\lambda_n \in B^+(0,\varepsilon_n)$ such that $d(x_n, K_1(x_n,\lambda_n)) \leq \varepsilon_n$ and $0 \in F_1(x_n,y,\lambda_n) + B^+(0,\varepsilon_n)e(x_n) \ \ \forall y \in K_2(x_n,\lambda_n)$, and since $x'_n \in \Omega_{(\text{QVIP})_\lambda}(\delta_n,\varepsilon_n)$, there exists $\lambda'_n \in B^+(0,\varepsilon_n)$ such that $d(x'_n, K_1(x_n,\lambda'_n)) \leq \varepsilon_n$ and $0 \in F_1(x'_n,y,\lambda'_n) + B^+(0,\varepsilon_n)e(x'_n) \ \ \forall y \in K_2(x_n,\lambda'_n)$. Clearly, $\lambda_n \to \lambda$ and $\lambda'_n \to \lambda$ as $n \to \infty$. Hence, $\{x_n\}$ and $\{x'_n\}$ are LP approximating solution sequences for $(\text{QVIP})_\lambda$ corresponding to λ_n, λ'_n, respectively. By the LP well-posed of $(\text{QVIP})_\lambda$, $\{x_n\}$ and $\{x'_n\}$ converge to the unique solution x_λ of $(\text{QVIP})_\lambda$, which is a contradiction to (3.11). This implies that (3.10). Conversely, suppose that condition (3.9) holds. Let $\lambda \in \Lambda$ be fixed. Let $\{\lambda_n\}$ be any sequence in Λ with $\lambda_n \to \lambda$ as $n \to \infty$. Suppose that $\{x_n\}$ is a LP approximating solution sequence for $(\text{QVIP})_\lambda$ corresponding to $\{\lambda_n\}$, then there exists a nonnegative sequence $\{\varepsilon_n\} \downarrow 0$ such that for each $n \in \mathbb{N}$, $d(x_n, K_1(x_n,\lambda)) \leq \varepsilon_n$, and $0 \in F_1(x_n,y,\lambda_n) + B^+(0,\varepsilon_n)e(x_n), \ \ \forall y \in K_2(x_n,\lambda)$. For each $n \in \mathbb{N}$, let $\delta_n = d'(\lambda_n,\lambda)$. Then, $\lambda_n \in B(\lambda,\delta_n)$ and $x_n \in \Omega_{(\text{QVIP})_\lambda}(\delta_n,\varepsilon_n)$ for each $n \in \mathbb{N}$, and $\delta_n \to 0$ as $n \to \infty$. It follows from (3.9) that $\{x_n\}$ is a Cauchy sequence and so it converges to a point $x \in X$. By similar arguments as in the proof of Lemma 3.4, we also deduce that x belongs to $S_{(\text{QVIP})_\lambda}$. Next, we will show that $(\text{QVIP})_\lambda$ has a unique solution. Suppose to the contrary, if $(\text{QVIP})_\lambda$ has two distinct solutions x_1 and x_2, it is easy to see that $x_1, x_2 \in \Omega_{(\text{QVIP})_\lambda}$ for all $\delta, \varepsilon > 0$. It follows that $0 < d(x_1,x_2) \leq \text{diam}(\Omega_{(\text{QVIP})_\lambda}(\delta,\varepsilon))$ which gives a contrdiction to (3.9). This implies that $(\text{QVIP})_\lambda$ has a unique solution. This completes the proof. □

The following example is given to illustrate the case that Theorem 3.1 is applicable.

Example 3.2. Let $E = Z = \mathbb{R}, X = [0,1]$ and $\Lambda = [0,1]$. For every $(x,y,\lambda) \in X \times X \times \Lambda$, let

$$e(x) = 1,\ K_1(x,\lambda) = \begin{cases} \left[0, \frac{1}{2}\right], & \text{if } \lambda \neq \frac{1}{2}, \\ [0,1], & \text{if } \lambda = \frac{1}{2}, \end{cases} \quad \text{and } K_2(x,\lambda) = \begin{cases} [0,1], & \text{if } \lambda \neq \frac{1}{2}, \\ \left[0, \frac{1}{2}\right], & \text{if } \lambda = \frac{1}{2}. \end{cases}$$

Define a set-valued mapping $F_1 : X \times X \times \Lambda \to 2^Z$ by $F_1(x,y,\lambda) = (-\infty, (\lambda+2)(y-x)]$. Obviously, it is to see that conditions of Theorem 3.1 are satisfied. For every $\lambda \in \Lambda, \text{diam}(\Omega_{(\text{QVIP})_\lambda}(\delta,\varepsilon)) \to 0$ as $(\delta,\varepsilon) \to (0,0)$. By Theorem 3.1, $(\text{QVIP})_\lambda$ is well-posed. □

Remark 3.3. We can not the supposed LP well-posedness in Theorem 3.1 by generalized LP well-posedness. Therefore, we have to employ the Kuratowski measure of noncompactness to study characterizations of the LP well-posedness in the generalized sense for (QVIP).

Theorem 3.2. *For* (QVIP)*, assume that E is complete and Λ is finite dimensional, K_1 is closed-valued and u.s.c., K_2 is l.s.c. and F_1 is closed. Then* (QVIP) *is LP well-posed in generalized the sense if and only if for every $\lambda \in \Lambda$, $\Omega_{(\text{QVIP})_\lambda}(\delta,\varepsilon) \neq \emptyset$, $\forall \delta, \varepsilon > 0$, and $\mu(\Omega_{(\text{QVIP})_\lambda}(\delta,\varepsilon)) \to 0$ as $(\delta,\varepsilon) \to (0,0)$.*

Proof. Suppose that (QVIP) LP well-posed in the generalized sense. Let $\lambda \in \Lambda$ be fixed. Then $S_{(\text{QVIP})_\lambda}$ is nonempty. Now we show that $S_{(\text{QVIP})_\lambda}$ is compact. Indeed, let $\{x_n\}$

be any sequence in $S_{(\mathrm{QVIP})_\lambda}$. Then $\{x_n\}$ is a LP approximating solution sequence for $(\mathrm{QVIP})_\lambda$. By the LP well-posedness in the generalized sense of (QVIP), $\{x_n\}$ has a subsequence which converges to a point of $S_{(\mathrm{QVIP})_\lambda}$. Thus $S_{(\mathrm{QVIP})_\lambda}$ is compact. Clearly, for each $\delta, \varepsilon > 0, S_{(\mathrm{QVIP})_\lambda} \subseteq \Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)$, and so $\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon) \neq \emptyset$. Now we will show that

$$\mu(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)) \to 0 \text{ as } (\delta, \varepsilon) \to (0, 0). \tag{3.12}$$

Observe that for every $\delta, \varepsilon > 0$, $\mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}) = H^*(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda})$, and $S_{(\mathrm{QVIP})_\lambda}$ is compact. Indeed, let $\{x_n\} := \{(x_n, \lambda_n)\}$ be arbitraly sequence in $S_{(\mathrm{QVIP})_\lambda}$. Then, it is clear that $\{x_n\}$ is a LP approximating sequence of (QVIP). Thus, it has a subsequence converging to a point in $S_{(\mathrm{QVIP})_\lambda}$. Therefore, $\mu(S_{(\mathrm{QVIP})_\lambda}) = 0$. Now for any $\alpha > 0$, there are finite sets $A_1^\alpha, A_2^\alpha, \ldots, A_{n_\alpha}^\alpha$ for some $n_\alpha \in \mathbb{N}$ such that $S_{(\mathrm{QVIP})_\lambda} \subseteq \bigcup_{k=1}^{n_\alpha} A_k^\alpha$ and $\operatorname{diam} A_k^\alpha \leq \alpha$, for all $k = 1, 2, \ldots, n_\alpha$. Next, for each $k \in \{1, 2, \ldots, n_\alpha\}$, we define the set $M_k^\alpha = \{z \in X : d(z, A_k^\alpha) \leq \mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda})\}$. We show that $\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon) \subseteq \bigcup_{k=1}^{n_\alpha} M_k^\alpha$. To this end, let $x \in \Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)$ be given. Thus, we have $d(x, S_{(\mathrm{QVIP})_\lambda}) \leq \mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda})$. As $S_{(\mathrm{QVIP})_\lambda} \subseteq \bigcup_{k=1}^{n_\alpha} A_k^\alpha$, we also get $d\left(x, \bigcup_{k=1}^{n_\alpha} A_k^\alpha\right) \leq d(x, S_{(\mathrm{QVIP})_\lambda}) \leq \mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda})$. Therefore, there exists $k_0 \in \{1, 2, \ldots, n_\alpha\}$ such that $d(x, A_{k_0}^\alpha) \leq \mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda})$, thereby yielding $x \in M_{k_0}^\alpha$. Therefore, we get the desired inclusion. Futhermore, we see that, for any $k \in \{1, 2, \ldots, n_\alpha\}$,

$$\operatorname{diam} M_k^\alpha \leq \alpha + 2\mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}). \tag{3.13}$$

Indeed, for any $y, y' \in M_k^\alpha$ and $m, m' \in A_k^\alpha$, $d(y, y') \leq d(y, m) + d(m, m') + d(m', y')$, which gives that $d(y, y') \leq \alpha + 2\mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda})$, which leads to the desired result (3.13). It follows from the definition of μ that $\mu(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)) \leq 2\mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}) + \alpha$, for all $\alpha > 0$. Therefore, we can conclude that

$$\mu(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)) \leq 2\mathcal{H}(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}) = 2H^*(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}).$$

To prove (3.8), it is sufficient to show that

$$H^*(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}) \to 0 \text{ as } (\delta, \varepsilon) \to (0, 0). \tag{3.14}$$

If (3.14) does not hold, then there exist $r > 0$, sequences $\{\delta_n\}$ and $\{\varepsilon_n\}$ of positive real numbers with $(\delta_n, \varepsilon_n) \to (0, 0)$ as $n \to \infty$ and sequence $\{x_n\}$ with $x_n \in \Omega_{(\mathrm{QVIP})_\lambda}(\delta_n, \varepsilon_n)$ for every $n \in \mathbb{N}$ such that

$$d(x_n, S_{(\mathrm{QVIP})_\lambda}) > r, \ \forall n \in \mathbb{N}. \tag{3.15}$$

For each $n \in \mathbb{N}$, since $x_n \in \Omega_\lambda(\delta_n, \varepsilon_n)$, there exists $\lambda_n \in B(\lambda, \delta_n)$ such that $d(x_n, K_1(x_n, \lambda_n)) \leq \varepsilon_n$ and $0 \in F(x_n, y, \lambda_n) + B^+(0, \varepsilon_n)e(x_n)$, $\forall y \in K_2(x_n, \lambda_n)$. Clearly $\lambda_n \to \lambda$ as $n \to \infty$. Hence $\{x_n\}$ is a LP approximating solution sequence for $(\mathrm{QVIP})_\lambda$ corresponding to $\{\lambda_n\}$. Then, by the LP well-posedness in the generalized sense of (QVIP), $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ which converges to some point of $S_{(\mathrm{QVIP})_\lambda}$. This contradicts (3.15), and so (3.14) holds. Therefore, (3.8) is proved. Conversely, suppose that condition (3.7) holds. We will show that (QVIP) is LP well-posed in generalized sense. Let $\lambda \in \Lambda$ be fixed. Thus, by Lemma 3.3 and Lemma 3.4, we have $\Omega_{(\mathrm{QVIP})_\lambda}$ is closed. Further, $S_{(\mathrm{QVIP})_\lambda} = \bigcap_{\delta, \varepsilon > 0} \Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)$. Since $\mu(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon)) \to 0$ as $(\delta, \varepsilon) \to (0, 0)$, by Lemma 2.2, $S_{(\mathrm{QVIP})_\lambda}$ is a nonempty compact subset of X and

$$H^*(\Omega_{(\mathrm{QVIP})_\lambda}(\delta, \varepsilon), S_{(\mathrm{QVIP})_\lambda}) \to 0 \text{ as } (\delta, \varepsilon) \to (0, 0). \tag{3.16}$$

Let $\{\lambda_n\}$ be any sequence in Λ with $\lambda_n \to \lambda$ as $n \to \infty$. Suppose that $\{x_n\}$ is a LP approximating solution sequence for $(\mathrm{QVIP})_\lambda$ corresponding to $\{\lambda_n\}$, then there exists a sequence $\{\varepsilon_n\}$ of positive real numbers with $\varepsilon_n \to 0$ such that, for each $n \in \mathbb{N}$, $d(x_n, K_1(x_n, \lambda_n)) \leq \varepsilon_n$ and $0 \in F_1(x_n, y, \lambda_n) + B^+(0, \varepsilon_n)e(x_n), \forall y \in K_2(x_n, \lambda_n)$. For each $n \in \mathbb{N}$, let $\delta_n = d(\lambda_n, \lambda)$. Then, $\lambda_n \in B(\lambda, \delta_n)$ and $x_n \in \Omega_{(\mathrm{QVIP})_\lambda}(\delta_n, \varepsilon_n)$ for every $n \in \mathbb{N}$, and $\delta_n \to 0$

as $n \to \infty$. It follows from (3.10) that $d(x_n, S_{(\text{QVIP})_\lambda}) \leq H^*(\Omega_{(\text{QVIP})_\lambda}(\delta_n, \varepsilon_n), S_{(\text{QVIP})_\lambda}) \to 0$ as $n \to \infty$. Since $S_{(\text{QVIP})_\lambda}$ is compact, for each $n \in \mathbb{N}$, there exists $\bar{x}_n \in S_{(\text{QVIP})_\lambda}$ such that $d(x_n, \bar{x}_n) = d(x_n, S_{(\text{QVIP})_\lambda}) \to 0$ as $n \to \infty$. By the compactness of $S_{(\text{QVIP})_\lambda}$, $\{\bar{x}_n\}$ has a subsequence $\{\bar{x}_{n_k}\}$ which converges to a point $\bar{x} \in S_{(\text{QVIP})_\lambda}$. Hence, the corresponding subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converges to $\bar{x}$. This implies that (QVIP) is LP well-posed in the genelized sense. This completes the proof. □

Remark 3.4. *Theorems 3.1, Theorems 3.2 generalizes Theorem 3.8, Theorems 3.11 of* [29], *respectively.*

By Theorems 3.1 and 3.2, we can get the following results.

Theorem 3.3. *For* (QVDP), *assume that E, K_1, K_2 as in Theorem 3.1 and F_2 is closed. Then* (QVDP) *is LP well-posed if and only if for every $\lambda \in \Lambda$,*

$$\Omega_{(\text{QVDP})_\lambda}(\delta, \varepsilon) \neq \emptyset, \forall \delta, \varepsilon > 0, \text{ and } \operatorname{diam}(\Omega_{(\text{QVDP})_\lambda}(\delta, \varepsilon)) \to 0 \text{ as } (\delta, \varepsilon) \to (0, 0).$$

Theorem 3.4. *For* (QVDP), *assume that E, K_1, K_2 as in Theorem 3.2 and F_2 is closed. Then* (QVDP) *is LP well-posed in generalized the sense if and only if for every $\lambda \in \Lambda$,*

$$\Omega_{(\text{QVDP})_\lambda}(\delta, \varepsilon) \neq \emptyset, \forall \delta, \varepsilon > 0, \text{ and } \mu(\Omega_{(\text{QVDP})_\lambda}(\delta, \varepsilon)) \to 0 \text{ as } (\delta, \varepsilon) \to (0, 0).$$

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DEPARTMENT OF MATHEMATICS
NARESUAN UNIVERSITY
FACULTY OF SCIENCE, PHITSANULOK, 65000 THAILAND
E-mail address: panatdaw@hotmail.com
E-mail address: rabianw@nu.ac.th