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Dedicated to Professor Yeol Je Cho on the occasion of his retirement

Existence and stability for a generalized differential mixed quasi-variational inequality

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ABSTRACT. In the present paper, we investigate a generalized differential mixed quasi-variational inequality consisting of a system of an ordinary differential equation and a generalized mixed quasi-variational inequality. By using an important result concerning the measurable selection, we prove the existence of Carathéodory weak solution to the generalized differential mixed quasi-variational inequality. Then, with the existence result, we establish two stability results for the generalized differential mixed quasi-variational inequality under different conditions, i.e., upper semicontinuity and lower semicontinuity of the Carathéodory weak solution with respect to the parameter, which is a perturbation of some mappings in the generalized mixed quasi-variational inequality.

1. INTRODUCTION

It is well known that variational inequality (VI), which has various kinds of generalizations such as quasi-variational inequality, mixed variational inequality and vector variational inequality etc., has been widely studied and applied into many research fields such as economics, optimization, mechanics and transportation etc. (see [1, 5, 7, 10, 15, 19, 20, 25, 26, 27, 28, 29, 30]). When an ordinary differential equation is involved, the system of variational inequality and ordinary differential equation is called differential variational inequality (DVI), which is introduced and studied by Pang and Stewart [16] in 2008. Recently, DVIs have attracted much attention of researchers in different research fields and many theoretical results, numerical algorithms and applications of DVIs have been studied by many authors. For the literature on the research of DVIs, we refer the reader to [14, 17, 18, 24] and the references therein.

The stability analysis of a VI or a DVI with perturbed data is concerned with the upper and lower semicontinuity, continuity, Lipschitz continuity or some kind of differentiability of its solution set. It can help in identifying sensitive parameters that should be obtained with relatively high accuracy, predicting the future changes of the equilibria as a result of the changes in the governing system, providing useful information for designing or planning various equilibrium systems. Consequently, the stability analysis of VIs and DVIs attracted the attention of many researchers in very early time. In 1986, Tobin studied the stability analysis for a VI when both the variational inequality function and the feasible region are perturbed in [21]. Khanh and Luu[6] studied the lower semicontinuity and upper semicontinuity of the solution sets and approximate solution sets of parametric multi-valued quasi-variational inequalities in topological vector spaces. Recently, Wang et al.[22] studied some upper semicontinuity and continuity results concerned with the

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Carathéodory weak solution set mapping for the differential set-valued mixed variational inequality. For more related research works, we can refer to [4, 8, 23] and the references therein.

Inspired by the above research on stability analysis of VI and DVI, in this paper, we study the existence and stability of solution to the following generalized differential mixed quasi-variational inequality (GDMQVI) in finite dimensional space:
Find $(x(t), u(t)) : [0, T] \to R^n \times R^n$ such that

$$(1.1) \qquad \begin{cases} \dot{x}(t) = f(t, x(t)) + B(t, x(t))u(t), \\ \langle y - h(u), G(t, x) + F(u)\rangle + p\varphi(y) - p\varphi(h(u)) \geq 0, \qquad \forall y \in K(u), \\ h(u) \in K(u), \\ x(0) = x_0. \end{cases}$$

where $\dot{x}(t) = \dfrac{dx}{dt}$ stands for the derivative of function x with respect to time variable t, $(f, B, G) : \Omega \to R^m \times R^{m\times n} \times R^n$ are given functions with $\Omega = [0, T] \times R^n$ while (h, F) are functions from R^n to R^n, $\varphi : R^n \to (-\infty, +\infty]$ is a functional, p is a positive real number, and $K : R^n \rightrightarrows R^n$ is a set-valued mapping such that $K(u) \subset R^n$ is a closed convex subset for each $u \in R^n$.

The following of this paper is as follows. First, in Section 2, we present some preliminaries. Then we give an existence result of Carathéodory weak solutions to the GDMQVI (1.1) in Section 3. At last, in Sect. 4, the upper semicontinuity and lower semicontinuity of Carathéodory weak solution set with respect to the perturbed data in the generalized mixed quasi-variational inequality is established.

2. PRELIMINARES

In this section, we introduce some basic notations and preliminary results, and present some definitions and hypotheses for the GDMQVI (1.1).

Definition 2.1. (see [12]) Let H be a real Hilbert space, and $g, A : H \to H$ be two single-valued mappings.

(i) A is said to be λ-strongly monotone on H if there exists a constant λ such that

$$\langle Ax - Ay, x - y\rangle \geq \lambda\|x - y\|^2, \quad \forall x, y \in H;$$

(ii) (A, g) is said to be a μ-strongly monotone couple on H if there exists a constant $\mu > 0$ such that

$$\langle Ax - Ay, g(x) - g(y)\rangle \geq \mu\|x - y\|^2, \quad \forall x, y \in H.$$

Definition 2.2. (see[2]) Let X, Y be two metric spaces. A set-valued mapping $F : X \rightrightarrows Y$ is called to be

(i) upper semicontinuous at $x_0 \in X$ iff for any neighborhood N of $F(x)$, there exists an $\eta > 0$, such that $F(x) \subset N$ for every $x \in B(x_0, \eta)$;

(i) lower semicontinuous at $x_0 \in X$ iff for any $y \in F(x_0)$ and for any sequence $x_n \in X$ converging to x_0, there exists a sequence of elements $y_n \in F(x_n)$ converging to y.

Definition 2.3. (see [12]) Let H be a real Hilbert space and $K : H \rightrightarrows H$ be a set-valued mapping such that $K(u)$ is a closed and convex subset of H for any $u \in H$. The generalized $f-$projection of $z \in H$ on the set $K(u)$ is defined by

$$P_{K(u)}z = arg \inf_{\xi\in K(u)} (\|z\|^2 - 2\langle z, \xi\rangle + \|\xi\|^2 + 2p\varphi(\xi)), \quad \forall z \in H.$$

Definition 2.4. A pair $(x(t), u(t))$ on $[0, T]$ is called a Carathéodory weak solution of the GDMQVI (1.1) if

(1) $h(u) \in K(u)$;
(2) $x(t)$ is absolutely continuous on $[0, T]$ and satisfies the differential equation in the GDMQVI (1.1) for almost all $t \in [0, T]$;
(3) $u \in L^2[0, T]$ satisfies the generalized mixed quasi-variational inequality in the GDMQVI (1.1) for every $t \in [0, T]$, where $L^2[0, T]$ denotes the set of all measurable functions $u : [0, T] \to R^n$ satisfying $\int_0^T \|u(t)\|^2 dt < +\infty$.

Lemma 2.1. ([13]). *Let $\varphi : R^n \to (-\infty, +\infty]$ be a proper lower semicontinuous convex functional. Suppose that the function $\varphi(u(\cdot))$ is integrable on $[0, T]$ for every $u \in L^2[0, T]$. Then*

$$\phi(u) = \int_0^T \varphi(u(t))dt, \quad u \in L^2[0, T]$$

is a proper lower semicontinuous convex functional.

Lemma 2.2. ([16]) *Let $\mathbb{F} : \Omega \rightrightarrows R^m$ be an upper semicontinuous set-valued mapping with nonempty closed convex values. Suppose that there exists a scalar $\rho^{\mathbb{F}} > 0$ satisfying*

$$\sup\{\| y \|: y \in \mathbb{F}(t, x)\} \leq \rho^{\mathbb{F}}(1+ \| x \|), \quad \forall (t, x) \in \Omega. \tag{2.2}$$

For every $x^0 \in R^n$, differential inclusion (DI) $\dot{x} \in \mathbb{F}(t, x), x(0) = x^0$ has a weak solution in the sense of Carathéodory.

Lemma 2.3. ([16]) *Let $H : \Omega \times R^m \to R^m$ be a continuous function and $U : \Omega \rightrightarrows R^n$ be a closed set-valued mapping such that for some constant $\eta_U > 0$,*

$$\sup_{u \in U(t,x)} \| u \| \leq \eta_U(1+ \| x \|), \quad \forall (t, x) \in \Omega.$$

Let $v : [0, T] \to R^m$ be a measurable function and $x : [0, T] \to R^m$ be a continuous function satisfying $v(t) \in H(t, x(t), U(t, x(t)))$ for almost all $t \in [0, T]$. There exists a measurable function $u : [0, T] \to R^n$ such that $u(t) \in U(t, x(t))$ and $v(t) = H(t, x(t), u(t))$ for almost all $t \in [0, T]$.

At the end of this section, we present the following two hypotheses (A) and (B), which hold for some functions in GDMQVI (1.1) in the rest of this paper.

(A) Suppose that the functions f, B and G are Lipschitz continuous functions on Ω with Lipschitz constants $L_f > 0$, $L_B > 0$ and $L_G > 0$, respectively.
(B) Suppose that the functions f and B are bounded on Ω with constants σ_f and σ_B. i.e., $\sigma_B \equiv \sup_{(t,x)\in\Omega} \| B(t, x) \| < \infty$ and $\sigma_f \equiv \sup_{(t,x)\in\Omega} \| f(t, x) \| < \infty$.

3. Existence of solutions to the GDMQVI

In this section, we prove the existence of Carathéodory weak solutions to the GDMQVI (1.1) by using Lemma 2.2 and Lemma 2.3. For this purpose, we define a set-valued mapping $\mathbb{F}(t, x) : \Omega \to 2^{R^m}$ as follows:

$$\mathbb{F}(t, x) \equiv \{f(t, x) + B(t, x)u : u \in S(G(t, x) + F(\cdot), h, \varphi, K(\cdot))\}, \tag{3.3}$$

where $S(G(t, x) + F(\cdot), h, \varphi, K(\cdot))$ denotes the solution set of the generalized mixed quasi-variational inequality in the GDMQVI (1.1). The following lemma presents some properties of the set-valued mapping $\mathbb{F}$ defined by (3.3) under the hypotheses (A) and (B).

Lemma 3.4. *Let the hypotheses (A) and (B) hold, the functions $h, F : R^n \to R^n$ and the set-valued mapping $K : R^n \rightrightarrows R^n$ be continuous on R^n, and for each $u \in R^n$, $\varphi : R^n \to R \bigcup \{+\infty\}$*

be proper and continuous on $K(u)$. Suppose that, for all $q \in G(\Omega)$, $S(q+F(\cdot),h,\varphi,K(\cdot))$ is nonempty and there exists a constant $\rho > 0$ such that

$$\sup\{\| u \|: u \in S(q+F,h,\varphi,K(\cdot))\} \leq \rho(1+ \| q \|). \tag{3.4}$$

Then there exists a constant $\rho^{\mathbb{F}} > 0$ such that

$$\sup\{\| y \|: y \in \mathbb{F}(t,x)\} \leq \rho^{\mathbb{F}}(1+ \| x \|),\ \forall (t,x) \in \Omega.$$

Moreover, $\mathbb{F}$ is upper semicontinuous and has closed-value on Ω.

Proof. The proof is very similar to the proof of Lemma 3.1 in [11]. We omit it here. □

Theorem 3.1. *Let the hypotheses (A) and (B), $F : R^n \to R^n$ be Lipschitz continuous with constants ρ_1 on R^n , h be a linear and inverse function with linear coefficient being ρ_2, and $\varphi : R^n \to R\bigcup\{+\infty\}$ be a proper, convex and continuous functional with $\varphi(u(\cdot))$ being integral for every integral function $u \in L^2[0,T]$. Suppose that*

- (i) *F is $\lambda - strongly\ monotone$ on R^n and (h,F) is $\mu - strongly\ monotone$ couple on R^n;*
- (ii) *there exists $k > 0$ such that $\|P_{K(u)}z - P_{K(y)}z\| \leq k\|u-y\|, \quad \forall u,y \in R^n, z \in \{v : v = h(u) - p(q+F(u)), u \in R^n, q \in G(\Omega)\}$.*
- (iii) *$({\rho_2}^2 - 2p\mu + p^2{\rho_1}^2)^{1/2} + p(1-2\lambda+{\rho_1}^2)^{1/2} < p-k$.*
- (iv) *$K : R^n \rightrightarrows R^n$ is a continuous set-valued mapping such that, for each $u \in R^n, K(u) \subset R^n$ is a closed convex set and K has a linear growth.*

Then the initial-value GDMQVI(1.1) has a Carathéodory weak solution.

Proof. By the Theorem 4.1 in [12], it follows from the assumptions (i), (ii) and (iii) that $S(q+F,h,\varphi,K(\cdot))$ is a nonempty singleton for every $q \in G(\Omega)$, which is assumed by $\{u\}$, and thus $h(u) \in K(u)$. Since h is linear, inverse and K has a linear growth, there exists a constant $c > 0$ such that for any $q \in G(\Omega)$,

$$\|u\| = \|h^{-1}h(u)\| \leq c\|h^{-1}\|(1+\|u\|).$$

Then by Lemma 3.4, the set-valued mapping $\mathbb{F}$, defined by (3.3), is an upper semicontinuous set-valued mapping with nonempty closed convex values on Ω and there exists a constant $\rho^{\mathbb{F}} > 0$ such that

$$\sup\{\| y \|: y \in \mathbb{F}(t,x)\} \leq \rho^{\mathbb{F}}(1+ \| x \|),\ \forall (t,x) \in \Omega.$$

This indicates by Lemma 2.2 that inclusion problem $DI : \dot{x} \in \mathbb{F}(t,x), x(0) = x_0$ has a weak solution x in the sense of Carathéodory, which implies that

$$\|x(t)\| \leq \|x_0\| + \int_0^t \rho^{\mathbb{F}}(1+\|x(s)\|)ds.$$

Moreover, by Gronwall's lemma,

$$\|x(t)\| \leq (\|x_0\| + \rho^{\mathbb{F}}T)e^{\rho^{\mathbb{F}}T}.$$

Now, we prove that $U(t,x) = S(G(t,x)+F,h,\varphi,K(\cdot))$ is closed on Ω. To this end, let $\{(t_n,x_n)\} \subset \Omega$ be a sequence converging to some vector $(t_0,x_0) \in \Omega$, and $\{u_n\} \subset U(t_n,x_n)$ converging to u_0. Thus, $h(u_n) \in K(u_n)$ and

$$\langle y - h(u_n), G(t_n,x_n) + F(u_n)\rangle + p\varphi(y) - p\varphi(h(u_n)) \geq 0, \qquad \forall y \in K(u_n). \tag{3.5}$$

Since h is continuous and K is upper semi-continuous on R^n, it follows $h(u_n) \in K(u_n)$ that $h(u_0) \in K(u_0)$. The lower semicontinuity of K implies that, for any $\hat{y} \in K(u_0)$, there exists $y_n \in K(u_n)$ such that $y_n \to \hat{y}$. This implies by (3.5) that

$$\langle y_n - h(u_n), G(t_n,x_n) + F(u_n)\rangle + p\varphi(y_n) - p\varphi(h(u_n)) \geq 0. \tag{3.6}$$

By letting $n \to \infty$ at both sides of the above inequality, we obtain that

$$\begin{aligned} & \langle \hat{y} - h(u_0), G(t_0, x_0) + F(u_0) \rangle + p\varphi(\hat{y}) - p\varphi(h(u_0)) \\ \geq \quad & \lim_{n\to\infty} [\langle y_n - h(u_n), G(t_n, x_n) + F(u_n) \rangle] + p\varphi(y_n) - p \liminf_{n\to\infty} \varphi(h(u_n)) \\ \geq \quad & 0. \end{aligned} \tag{3.7}$$

This means that $u_0 \in U(t_0, x_0)$ and thus $U(t, x)$ is closed on Ω. Then, by Lemma 2.3 we know GDMQVI (1.1) admits a Carathéodory weak solution. □

4. Stability for the GDMQVI

In this section, we aim to study the stability of solutions to the GDMQVI (1.1). For this purpose, we consider the parametric GDMQVI, which is denoted by PGDMQVI, as follows:

$$\begin{cases} \dot{x}(t) = f(t, x(t)) + B(t, x(t))u(t), \\ \langle y - h(u), G(t, x) + F(u, z) \rangle + p\varphi(y) - p\varphi(h(u)) \geq 0, \qquad \forall y \in K(u, z), \\ h(u) \in K(u, z), \\ x(0) = x_0, \end{cases} \tag{4.8}$$

where, with (Z, d) being a metric space, $F : R^n \times Z \to R^n$ and $K : R^n \times Z \rightrightarrows R^n$ are the perturbed mappings of the corresponding mappings in the GDMQVI (1.1) respectively. For easy of writing, we denote by $SD(z)$ the Carath$\acute{e}$odory weak solution of the GDMQVI (4.8) and the set of all u is denoted by $SD_u(z)$.

Theorem 4.2. *Let the hypotheses (A) and (B) hold, $h : R^n \to R^n$ be linear and inverse, $F : R^n \times Z \to R^n$ and $K : R^n \times Z \rightrightarrows R^n$ be continuous mappings such that $K(\cdot, z)$ have a linear growth for every $z \in Z$, $\varphi : R^n \to R \bigcup \{+\infty\}$ be a proper, convex and continuous functional with $\varphi(u(t))$ being integral for every $u \in L^2[0, T]$, and $z_0 \in Z$ be a given point. Suppose that*

- (i) *there exists a neighborhood $U(z_0)$ of z_0 such that, for any $z \in U(z_0)$ and $q \in G(\Omega)$, $S(q + F(\cdot, z), h, \varphi, K(\cdot, z))$ is a nonempty singleton;*
- (ii) *K is closed on $R^n \times \{z_0\}$ and $K(R^n \times Z)$ is a bounded set;*
- (iii) *$SD_u(z)$ is uniformly compact at z_0.*

Then $SD(z)$ is upper semicontinuous at $z_0 \in Z$.

Proof. By Theorem 3.1, it follows from assumptions (i) and (ii) that $SD(z)$ is nonempty for any $z \in U(z_0)$. Now, we prove the assertion. We assume the contrary holds for the sake of contradiction. Then there exists an open set N containing $SD(z_0)$ such that, for every sequence $z_n \in U(z_0)$ with $z_n \to z_0$, there exists $(x_n, u_n) \in SD(z_n)$ but $(x_n, u_n) \notin N$, for every n. Since $(x_n, u_n) \in SD(z_n)$, we get that

(1) for any $0 \leq s \leq t \leq T$, $x_n(t) - x_n(s) = \int_s^t [f(\tau, x_n(\tau)) + B(\tau, x_n(\tau))u_n(\tau)]d\tau$;

(2) $h(u_n(t)) \in K(u_n(t), z_n)$ and for any $\hat{y} \in K(u_n(t), z_n)$,

$$\langle \hat{y} - h(u_n(t)), G(t, x_n(t)) + F(u_n(t)) \rangle + p\varphi(\hat{y}) - p\varphi(h(u_n(t))) \geq 0;$$

(3) $x_n(0) = x_0$.

Since h is linear and inverse and $K(R^n \times Z)$ is bounded, there exists a constant $C > 0$ such that, for any n and $t \in [0, T]$, $\|u_n(t)\| < C$. By the hypotheses (A), (B) and the boundedness of $K(R^n \times Z)$, it follows from (1) that $\{x_n\}$ is uniformly bounded with the norm $\|x\|_1 = \sup_{t\in[0,T]} \|x(t)\|$ and there exists a constant $M > 0$ such that, for any n,

$$\|x_n(t) - x_n(s)\| \leq M|t - s|.$$

Thus, from Arzelá-Ascoli theorem, we know $\{x_n\}$ has a subsequence, denoted by $\{x_n\}$, such that $x_n \to \hat{x}$. Meanwhile, since $SD_u(z)$ is uniformly compact at z_0, there exists a subsequence of $\{u_n\}$, denoted by $\{u_n\}$, such that $u_n \to \hat{u}$. This means $(x_n, u_n) \to (\hat{x}, \hat{u}) \notin N$. Therefore, for any $0 \leq s \leq t \leq T$, (1) and (3) imply that

$$\hat{x}(t) - \hat{x}(s) = \int_s^t [f(\tau, \hat{x}(\tau)) + B(\tau, x(\tau))\hat{u}(\tau)]d\tau \tag{4.9}$$

$$\hat{x}(0) = x_0. \tag{4.10}$$

By the assumption (ii), i.e., K is closed on $R^n \times \{z_0\}$, we get from (2) that

$$h(\hat{u}(t)) \in K(\hat{u}(t), z_0). \tag{4.11}$$

Moreover, since K is lower semicontinuous, for any $y_0 \in K(\hat{u}(t), z_0)$, there exists a sequence $\{y_n\} \subset K(u_n(t), z_n)$ such that $y_n \to y_0$. It follows from (2) that, for any $y_0 \in K(\hat{u}(t), z_0)$,

$$\begin{aligned}&\langle y_0 - h(\hat{u}(t)), G(t, \hat{x}(t)) + F(\hat{u}(t), z_0)\rangle + p\varphi(y_0) - p\varphi(h(\hat{u}(t))\\ &\geq \lim_{n\to\infty} \langle y_n - h(u_n(t)), G(t, x_n(t)) + F(u_n(t), z_n)\rangle + p \liminf_{n\to\infty} [\varphi(y_n) - p\varphi(h(u_n(t)))]\end{aligned}$$

$$\geq 0. \tag{4.12}$$

It follows (4.9)-(4.12) that $(\hat{x}, \hat{u}) \in SD(z_0)$, which contradicts to $(\hat{x}, \hat{u}) \notin N$. □

Theorem 4.3. *Let the hypotheses (A) and (B) hold for the mappings (f, G, B), $h = I : R^n \to R^n$ be the identity function, $z_0 \in Z$ be a given point, $F : R^n \times Z \to R^n$ and $K : R^n \times Z \rightrightarrows R^n$ be continuous mappings such that $F(\cdot, z_0)$ is λ-strongly monotone with $\lambda > 0$ and $K(\cdot, z)$ have a linear growth for every $z \in Z$, and $\varphi : R^n \to R \bigcup \{+\infty\}$ be a proper, convex and continuous functional with $\varphi(u(t))$ being integral for every $u \in L^2[0, T]$. Suppose that*

- (i) *there exists a neighborhood $U(z_0)$ of z_0 such that $SD(z)$ is nonempty for any $z \in U(z_0)$;*
- (ii) *K is closed on $R^n \times \{z_0\}$, $K(R^n \times Z)$ is bounded, and $K(\hat{u}(t), z_0) = K(\bar{u}(t), z_0)$ for any $\hat{u}, \bar{u} \in SD_u(z_0)$;*
- (iii) *$SD_u(z)$ is uniformly compact at z_0;*
- (iv) *for any $(\hat{x}, \hat{u}) \in SD(z_0)$,*

$$\langle y - \hat{u}(t), G(t, \hat{x}(t)) + F(\hat{u}(t), z_0)\rangle + p\varphi(y) - p\varphi(\hat{u}(t)) > 0, \quad \forall y \in K(\hat{u}(t), z_0)\backslash\{\hat{u}(t)\}.$$

Then $SD(z)$ is lower semicontinuous at z_0.

Proof. Suppose, on the contrary, that $SD(z)$ is not lower semicontinuous at z_0. Then there exists a sequence $\{z_n\}$ in Z with $z_n \to z_0$, and $(\hat{x}, \hat{u}) \in SD(z_0)$ such that, for every sequence $(x_n, u_n) \in SD(z_n)$, $(x_n, u_n) \nrightarrow (\hat{x}, \hat{u})$. Since $SD_u(z)$ is uniformly compact at z_0, there exists a subsequence of $\{u_n\}$, denoted by $\{u_{n_k}\}$, such that $u_{n_k} \to \bar{u}$. By similar arguments in proof of Theorem 4.2, it is easy to obtain that $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ such that $x_{n_k} \to \bar{x}$ and $(\bar{x}, \bar{u}) \in SD(z_0)$. Thus, for any $t \in [0, T]$,

$$\bar{x}(t) = x_0 + \int_0^t [f(\tau, \bar{x}(\tau)) + B(\tau, \bar{x}(\tau))\bar{u}(\tau)]d\tau, \tag{4.13}$$

$$\hat{x}(t) = x_0 + \int_0^t [f(\tau, \hat{x}(\tau)) + B(\tau, \hat{x}(\tau))\hat{u}(\tau)]d\tau. \tag{4.14}$$

And, by the contradiction assumption, $(\bar{x}, \bar{u}) \neq (\hat{x}, \hat{u})$. This means that $\bar{x} \neq \hat{x}$ or $\bar{u} \neq \hat{u}$. Note that if $\bar{x} \neq \hat{x}$, it is easy to get from (4.13) and (4.14) that $\bar{u} \neq \hat{u}$. If $\hat{u} \neq \bar{u}$, it follows from the assumption (iv) that, for all $y_1 \in K(\hat{u}(t), z_0)\backslash\{\hat{u}(t)\}$,

$$\langle y_1 - \hat{u}(t), G(t, \hat{x}(t)) + F(\hat{u}(t), z_0)\rangle + p\varphi(y_1) - p\varphi(\hat{u}(t)) > 0 \tag{4.15}$$

and, for all $y_2 \in K(\bar{u}(t), z_0)\backslash\{\bar{u}(t)\}$,

$$\langle y_2 - \bar{u}(t), G(t, \bar{x}(t)) + F(\bar{u}(t), z_0)\rangle + p\varphi(y_2) - p\varphi(\bar{u}(t)) > 0. \tag{4.16}$$

Letting $y_1 = \bar{u}(t)$ in (4.15) and $y_2 = \hat{u}(t)$ in (4.16), we get by adding the obtained inequalities that

$$\langle \bar{u}(t) - \hat{u}(t), F(\bar{u}(t), z_0) - F(\hat{u}(t), z_0)\rangle < \langle \bar{u}(t) - \hat{u}(t), G(t, \hat{x}(t)) - G(t, \bar{x}(t))\rangle.$$

Since $F : R^n \times Z \to R^n$ is λ-strongly monotone on $R^n \times \{z_0\}$ and G is Lipschitz continuous function on Ω with Lipschitz constants $L_G > 0$, we get from the above inequality that

$$\|\bar{u}(t) - \hat{u}(t)\| < \frac{L_G}{\lambda}\|\bar{x}(t) - \hat{x}(t)\|.$$

Thus, it follows from (4.13) and (4.14) that there exists a constant $C_1 > 0$ such that

$$\|\bar{x}(t) - \hat{x}(t)\| < C_1 \int_0^t \|\bar{x}(s) - \hat{x}(s)\| ds.$$

By applying the Gronwall inequality, it is easy to get that $\|\bar{x}(t) - \hat{x}(t)\| < 0$, which is a contradiction. □

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