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Dedicated to Prof. Emeritus Mihail Megan on the occasion of his 75^{th} anniversary

Hyers-Ulam stability of hom-derivations in Banach algebras

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ABSTRACT. In this work, we prove the Hyers–Ulam stability of hom-derivations in complex Banach algebras, associated with the additive (s_1, s_2)-functional inequality:

$$\begin{aligned} \|f(a+b)-f(a)-f(b)\| \quad &\leq \quad \|s_1\left(f(a+b)+f(a-b)-2f(a)\right)\| \\ &\quad + \left\|s_2\left(2f\left(\frac{a+b}{2}\right)-f(a)-f(b)\right)\right\|, \end{aligned} \tag{0.1}$$

where s_1 and s_2 are fixed nonzero complex numbers with $\sqrt{2}|s_1| + |s_2| < 1$.

1. INTRODUCTION AND PRELIMINARIES

The stability problem of functional equations originated from a question of Ulam [28] concerning the stability of group homomorphisms. These question form is the object of the stability theory. If the answers is affirmative, we say that the functional equation for homomorphism is stable. Hyers [11] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki [1] for additive mappings and by Rassias [24] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Rassias theorem was obtained by Găvruta [10] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias' approach. Park [17, 18] defined additive ρ-functional inequalities and proved the Hyers-Ulam stability of the additive ρ-functional inequalities in Banach spaces and non-Archimedean Banach spaces. The stability problems of various functional equations and functional inequalities have been extensively investigated by a number of authors (see [8, 9, 13, 14, 15, 16, 29]).

Applications of stability theory of functional equations for the proof of new fixed point theorems with applications were the first to furnished by Isac and Rassias [12] in 1996. The stability problems of several functional equations by using fixed-point methods have been extensively investigated by a number of authors, see [4, 5, 7, 19, 23]. For Hyers-Ulam stability of some integral and differential equations, see [26, 27] while for Hyers-Ulam stability of the fixed point problems in metric spaces see [2, 22, 25].

Recently, Park [20] proved the Hyers-Ulam stability of the additive (s_1, s_2)-functional inequality (0.1) in complex Banach spaces by using the fixed point method and the direct method. Next, Park et al. [21] solved the additive s-functional inequality:

$$\|f(a+b)-f(a)-f(b)\| \leq \|s(f(a-b)-f(a)-f(-b))\|,$$

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where s is a fixed nonzero complex number with $|s| < 1$. Using the fixed point method and the direct method, they proved the Hyers–Ulam stability of the additive s-functional inequality in complex Banach spaces. Also, they presented the Hyers–Ulam stability of hom-derivations in complex Banach algebras.

To obtain the desired results, the following definition is needed to be used later.

Definition 1.1. [21, Definition 1.1] *Let $\mathcal{A}$ be a complex Banach algebra and $G : \mathcal{A} \to \mathcal{A}$ be a homomorphism. A $\mathbb{C}$-linear mapping $F : \mathcal{A} \to \mathcal{A}$ is called a hom-derivation on $\mathcal{A}$ if F satisfies*

$$F(ab) = F(a)G(b) + F(a)G(b)$$

for all $a, b \in \mathcal{A}$.

Now, we recall a fundamental result in fixed point theory.

Theorem 1.1. [3, 6] *Let $(\mathcal{X}, d)$ be a complete generalized metric space and $\mathcal{J} : \mathcal{X} \to \mathcal{X}$ be a strictly contractive mapping with Lipschitz constant $\alpha < 1$. Then for each given element $x \in \mathcal{X}$, either*

$$d(\mathcal{J}^n x, \mathcal{J}^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $(\mathcal{J}^n x, \mathcal{J}^{n+1} x) < \infty, \qquad \forall n \geq n_0$;
(2) *the sequence $\{\mathcal{J}^n x\}$ converges to a fixed point y^* of $\mathcal{J}$;*
(3) *y^* is the unique fixed point of $\mathcal{J}$ in the set $Y = \{y \in \mathcal{X} \mid d(\mathcal{J}^{n_0} x, y) < \infty\}$;*
(4) *$d(y, y^*) \leq \frac{1}{1-\alpha} d(y, \mathcal{J} y)$ for all $y \in Y$.*

This paper is organized as follows: In Sections 2 and 3, using the direct method and using the fixed point method, we prove the Hyers–Ulam stability of hom-derivations in Banach algebras, associated with the additive (s_1, s_2)-functional inequality (0.1).

2. Hom-derivations in Banach algebras: a direct method

In this section, we prove the Hyers–Ulam stability of hom-derivations in Banach algebras, associated with the additive (s_1, s_2)-functional inequality (0.1) by using the direct method.

Theorem 2.2. *Let $\varphi : \mathcal{A}^2 \to [0, \infty)$ be a function satisfying*

$$\sum_{j=1}^{\infty} 4^j \varphi\left(\frac{a}{2^j}, \frac{b}{2^j}\right) < \infty \tag{2.2}$$

for all $a, b \in \mathcal{A}$ and let $f, g : \mathcal{A} \to \mathcal{A}$ be mappings satisfying

$$\begin{aligned} \|f(\lambda(a+b)) - \lambda(f(a) + f(b))\| &\leq \|s_1(f(a+b) + f(a-b) - 2f(a))\| \\ &\quad + \left\|s_2\left(2f\left(\frac{a+b}{2}\right) - f(a) - f(b)\right)\right\| + \varphi(a, b), \end{aligned} \tag{2.3}$$

$$\begin{aligned} \|g(\lambda(a+b)) - \lambda(g(a) + g(b))\| &\leq \|s_1(g(a+b) + g(a-b) - 2g(a))\| \\ &\quad + \left\|s_2\left(2g\left(\frac{a+b}{2}\right) - g(a) - g(b)\right)\right\| + \varphi(a, b), \end{aligned} \tag{2.4}$$

$$\varphi(a, b) \geq \|g(ab) - g(a)g(b)\|, \tag{2.5}$$

$$\varphi(a, b) \geq \|f(ab) - f(a)g(b) - g(a)f(b)\|, \tag{2.6}$$

and $f(0) = g(0) = 0$ for all $a, b \in \mathcal{A}$ and all $\lambda \in \mathbb{T}^1 := \{\mu \in \mathbb{C} : |\mu| < 1\}$. Then, there exist a unique homomorphism $G : \mathcal{A} \to \mathcal{A}$ and a unique hom-derivation $F : \mathcal{A} \to \mathcal{A}$ such that

$$\|f(a) - F(a)\| \leq \frac{1}{2(1-|s_1|)}\Psi(a,a), \tag{2.7}$$

$$\|g(a) - G(a)\| \leq \frac{1}{2(1-|s_1|)}\Psi(a,a), \tag{2.8}$$

$$F(ab) = F(a)G(b) + G(a)F(b), \tag{2.9}$$

where

$$\Psi(a,b) := \sum_{j=1}^{\infty} 2^j \varphi\left(\frac{a}{2^j}, \frac{b}{2^j}\right) \tag{2.10}$$

for all $a, b \in \mathcal{A}$.

Proof. Let $\lambda = 1$ in (2.3) and (2.4). By Theorem 3.1 in [20], there are unique additive mappings $F, G : \mathcal{A} \to \mathcal{A}$ satisfying (2.7) and (2.8), respectively, which are given by

$$G(a) = \lim_{k\to\infty} 2^k g\left(\frac{a}{2^k}\right)$$

and

$$F(a) = \lim_{k\to\infty} 2^k f\left(\frac{a}{2^k}\right)$$

for all $a \in \mathcal{A}$. Letting $b = 0$ in (2.3), we get

$$\|f(\lambda a) - \lambda f(a)\| \leq \left\|s_2\left(2f\left(\frac{a}{2}\right) - f(a)\right)\right\| + \varphi(a, 0)$$

for all $\lambda \in \mathbb{T}^1$ and all $a \in \mathcal{A}$. So

$$\begin{aligned}
\|F(\lambda a) - \lambda F(a)\| &= \lim_{k\to\infty} 2^k \left\|f\left(\lambda\frac{a}{2^k}\right) - \lambda f\left(\frac{a}{2^k}\right)\right\| \\
&\leq \lim_{k\to\infty} 2^k \left\|s_2\left(2f\left(\frac{a}{2^{k+1}}\right) - f\left(\frac{a}{2^k}\right)\right)\right\| + \lim_{k\to\infty} 2^k\varphi\left(\frac{a}{2^k}, 0\right) \\
&= 0.
\end{aligned}$$

Hence

$$F(\lambda x) = \lambda F(x)$$

for all $\lambda \in \mathbb{T}^1$ and all $a \in \mathcal{A}$. So, the mapping $F : \mathcal{A} \to \mathcal{A}$ is $\mathbb{C}$-linear. Similarly, one can show that the additive mapping $G : \mathcal{A} \to \mathcal{A}$ is $\mathbb{C}$-linear. For each $a, b \in \mathcal{A}$, it follows from (2.5) that

$$\begin{aligned}
\|G(ab) - G(a)G(b)\| &= \lim_{k\to\infty} 4^k \left\|g\left(\frac{ab}{2^k \cdot 2^k}\right) - g\left(\frac{a}{2^k}\right) g\left(\frac{b}{2^k}\right)\right\| \\
&\leq \lim_{k\to\infty} 4^k\varphi\left(\frac{a}{2^k}, \frac{b}{2^k}\right) \\
&= 0.
\end{aligned}$$

So

$$G(xy) = G(x)G(y)$$

for all $a, b \in \mathcal{A}$. Thus, the $\mathbb{C}$-linear mapping $G : \mathcal{A} \to \mathcal{A}$ is a homomorphism satisfying (2.8). For each $a, b \in \mathcal{A}$, it follows from (2.6) that

$$\begin{aligned} & \|F(ab) - F(a)G(b) - G(a)F(b)\| \\ = & \lim_{k\to\infty} 4^k \left\| f\left(\frac{ab}{2^k \cdot 2^k}\right) - f\left(\frac{a}{2^k}\right) g\left(\frac{b}{2^k}\right) - g\left(\frac{a}{2^k}\right) f\left(\frac{b}{2^k}\right) \right\| \\ \le & \lim_{k\to\infty} 4^k \varphi\left(\frac{a}{2^k}, \frac{b}{2^k}\right) \\ = & \ 0. \end{aligned}$$

Hence, the $\mathbb{C}$-linear mapping $F : \mathcal{A} \to \mathcal{A}$ is a hom-derivation satisfying (2.7) and (2.9). □

Corollary 2.1. *Let $r > 1$ and θ be nonnegative real numbers, and let $f, g : \mathcal{A} \to \mathcal{A}$ be mappings satisfying*

$$\begin{aligned} \|f(\lambda(a+b)) - \lambda(f(a)+f(b))\| \le & \ \|s_1(f(a+b)+f(a-b)-2f(a))\| \\ & + \left\| s_2\left(2f\left(\frac{a+b}{2}\right) - f(a) - f(b)\right)\right\| \\ & + \theta\left(\|a\|^r + \|b\|^r\right), \end{aligned} \quad (2.11)$$

$$\begin{aligned} \|g(\lambda(a+b)) - \lambda(g(a)+g(b))\| \le & \ \|s_1(g(a+b)+g(a-b)-2g(a))\| \\ & + \left\| s_2\left(2g\left(\frac{a+b}{2}\right) - g(a) - g(b)\right)\right\| \\ & + \theta\left(\|a\|^r + \|b\|^r\right), \end{aligned} \quad (2.12)$$

$$\theta\left(\|a\|^r + \|b\|^r\right) \ge \|g(ab) - g(a)g(b)\|, \quad (2.13)$$

$$\theta\left(\|a\|^r + \|b\|^r\right) \ge \|f(ab) - f(a)g(b) - g(a)f(b)\|, \quad (2.14)$$

and $f(0) = g(0) = 0$ for all $a, b \in \mathcal{A}$ and all $\lambda \in \mathbb{T}^1$. Then, there exist a unique homomorphism $G : \mathcal{A} \to \mathcal{A}$ and a unique hom-derivation $F : \mathcal{A} \to \mathcal{A}$ satisfying (2.9) *and*

$$\|f(a) - F(a)\| \le \frac{2\theta}{(1-|s_1|)(2^r-2)}\|a\|^r, \quad (2.15)$$

$$\|g(a) - G(a)\| \le \frac{2\theta}{(1-|s_1|)(2^r-2)}\|a\|^r \quad (2.16)$$

for all $a \in \mathcal{A}$.

Proof. The proof follows from Theorem 2.2 by taking $\varphi(a,b) = \theta\left(\|a\|^r + \|b\|^r\right)$ for all $a, b \in \mathcal{A}$. □

Theorem 2.3. *Let $\varphi : \mathcal{A}^2 \to [0, \infty)$ be a function satisfying*

$$\Psi(a,b) := \sum_{j=0}^{\infty} \frac{1}{2^j}\varphi\left(2^j a, 2^j b\right) < \infty \quad (2.17)$$

for all $a, b \in \mathcal{A}$ and let $f, g : \mathcal{A} \to \mathcal{A}$ be mappings satisfying $f(0) = g(0) = 0$ and (2.3)-(2.6). *Then, there exist a unique homomorphism $G : \mathcal{A} \to \mathcal{A}$ satisfying* (2.8) *and a unique hom-derivation $F : \mathcal{A} \to \mathcal{A}$ satisfying* (2.7) *and* (2.9).

Proof. Let $\lambda = 1$ in (2.3) and (2.4). By Theorem 3.3 in [20], there are unique additive mappings $F, G : \mathcal{A} \to \mathcal{A}$ satisfying (2.7) and (2.8), respectively, which are given by

$$G(a) = \lim_{k\to\infty} \frac{1}{2^k} g\left(2^k a\right)$$

and

$$F(a)=\lim_{k\to\infty}\frac{1}{2^k}f\left(2^k a\right)$$

for all $a\in\mathcal{A}$. The rest of the proof is similar to the proof of Theorem 2.2. □

Corollary 2.2. *Let $r<1$ and θ be positive real numbers, and let $f,g:\mathcal{A}\to\mathcal{A}$ be mappings satisfying $f(0)=g(0)=0$ and* (2.11)-(2.14). *Then, there exist a unique homomorphism $G:\mathcal{A}\to\mathcal{A}$ and a unique hom-derivation $F:\mathcal{A}\to\mathcal{A}$ satisfying* (2.9) *and*

$$\|f(a)-F(a)\|\leq\frac{2\theta}{(1-|s_1|)(2-2^r)}\|a\|^r, \tag{2.18}$$

$$\|g(a)-G(a)\|\leq\frac{2\theta}{(1-|s_1|)(2-2^r)}\|a\|^r \tag{2.19}$$

for all $a\in\mathcal{A}$.

Proof. The proof follows from Theorem 2.3 by taking $\varphi(a,b)=\theta\left(\|a\|^r+\|b\|^r\right)$ for all $a,b\in\mathcal{A}$. □

3. Hom-derivations in Banach algebras: a fixed point method

In this section, we present the Hyers-Ulam stability of hom-derivations in Banach algebras, associated to the additive (s_1,s_2)-functional inequality (0.1) by using the fixed point method.

Theorem 3.4. *Let $\varphi:\mathcal{A}^2\to[0,\infty)$ be a function such that there exists an $L<1$ with*

$$\varphi\left(\frac{a}{2},\frac{b}{2}\right)\leq\frac{L}{4}\varphi(a,b)\leq\frac{L}{2}\varphi(a,b) \tag{3.20}$$

for all $a,b\in\mathcal{A}$ and let $f,g:\mathcal{A}\to\mathcal{A}$ be mappings satisfying $f(0)=g(0)=0$ and (2.3)-(2.6). *Then, there exist a unique homomorphism $G:\mathcal{A}\to\mathcal{A}$ and a unique hom-derivation $F:\mathcal{A}\to\mathcal{A}$ satisfying* (2.9) *and*

$$\|f(a)-F(a)\| \leq \frac{L}{2(1-L)(1-|s_1|)}\varphi(a,a), \tag{3.21}$$

$$\|g(a)-G(a)\| \leq \frac{L}{2(1-L)(1-|s_1|)}\varphi(a,a) \tag{3.22}$$

for all $a\in\mathcal{A}$.

Proof. Let $\lambda=1$ in (2.3) and (2.4). By Theorem 2.2 in [20], there are unique additive mappings $F,G:\mathcal{A}\to\mathcal{A}$ satisfying (3.21) and (3.22), respectively, which are given by

$$G(a)=\lim_{k\to\infty}2^k g\left(\frac{a}{2^k}\right)$$

and

$$F(a)=\lim_{k\to\infty}2^k f\left(\frac{a}{2^k}\right)$$

for all $a\in\mathcal{A}$. Letting $b=0$ in (2.3), we get

$$\|f(\lambda a)-\lambda f(a)\| \leq \left\|s_2\left(2f\left(\frac{a}{2}\right)-f(a)\right)\right\|+\varphi(a,0)$$

for all $\lambda \in \mathbb{T}^1$ and all $a \in \mathcal{A}$. So

$$\begin{aligned} \|F(\lambda a) - \lambda F(a)\| &= \lim_{k\to\infty} 2^k \left\| f\left(\lambda \frac{a}{2^k}\right) - \lambda f\left(\frac{a}{2^k}\right) \right\| \\ &\le \lim_{k\to\infty} 2^k \left\| s_2\left(2f\left(\frac{a}{2^{k+1}}\right) - f\left(\frac{a}{2^k}\right)\right) \right\| + \lim_{k\to\infty} 2^k \varphi\left(\frac{a}{2^k}, 0\right) \\ &\le \lim_{k\to\infty} \left(\frac{L}{2}\right)^k \varphi(a, 0) \\ &= 0. \end{aligned}$$

Hence

$$F(\lambda x) = \lambda F(x)$$

for all $\lambda \in \mathbb{T}^1$ and all $a \in \mathcal{A}$. So, the mapping $F : \mathcal{A} \to \mathcal{A}$ is $\mathbb{C}$-linear. Similarly, one can show that the additive mapping $G : \mathcal{A} \to \mathcal{A}$ is $\mathbb{C}$-linear. For each $a, b \in \mathcal{A}$, it follows from (2.5) that

$$\begin{aligned} \|G(ab) - G(a)G(b)\| &= \lim_{k\to\infty} 4^k \left\| g\left(\frac{ab}{2^k \cdot 2^k}\right) - g\left(\frac{a}{2^k}\right) g\left(\frac{b}{2^k}\right) \right\| \\ &\le \lim_{k\to\infty} 4^k \varphi\left(\frac{a}{2^k}, \frac{b}{2^k}\right) \\ &\le \lim_{k\to\infty} L^k \varphi(a, b) \\ &= 0. \end{aligned}$$

So

$$G(xy) = G(x)G(y)$$

for all $a, b \in \mathcal{A}$. Thus, the $\mathbb{C}$-linear mapping $G : \mathcal{A} \to \mathcal{A}$ is a homomorphism satisfying (3.22). For each $a, b \in \mathcal{A}$, it follows from (2.6) that

$$\begin{aligned} &\|F(ab) - F(a)G(b) - G(a)F(b)\| \\ &= \lim_{k\to\infty} 4^k \left\| f\left(\frac{ab}{2^k \cdot 2^k}\right) - f\left(\frac{a}{2^k}\right) g\left(\frac{b}{2^k}\right) - g\left(\frac{a}{2^k}\right) f\left(\frac{b}{2^k}\right) \right\| \\ &\le \lim_{k\to\infty} 4^k \varphi\left(\frac{a}{2^k}, \frac{b}{2^k}\right) \\ &\le \lim_{k\to\infty} L^k \varphi(a, b) \\ &= 0. \end{aligned}$$

Hence, the $\mathbb{C}$-linear mapping $F : \mathcal{A} \to \mathcal{A}$ is a hom-derivation satisfying (2.9) and (3.21). □

Corollary 3.3. *Let $r > 1$ and θ be nonnegative real numbers, and let $f, g : \mathcal{A} \to \mathcal{A}$ be mappings satisfying $f(0) = g(0) = 0$ and* (2.11)-(2.14). *Then, there exist a unique homomorphism $G : \mathcal{A} \to \mathcal{A}$ satisfying* (2.16) *and a unique hom-derivation $F : \mathcal{A} \to \mathcal{A}$ satisfying* (2.9) *and* (2.15).

Proof. The proof follows from Theorem 3.4 by taking $\varphi(a, b) = \theta(\|a\|^r + \|b\|^r)$ for all $a, b \in \mathcal{A}$. Choosing $L = 2^{1-r}$, we obtain the desired result. □

Theorem 3.5. *Let $\varphi : \mathcal{A}^2 \to [0, \infty)$ be a function such that there exists an $L < 1$ with*

$$\varphi(a, b) \le 2L\varphi\left(\frac{a}{2}, \frac{b}{2}\right) \tag{3.23}$$

for all $a, b \in \mathcal{A}$ and let $f, g : \mathcal{A} \to A$ be mappings satisfying $f(0) = g(0) = 0$ and (2.3)-(2.6). *Then, there exist a unique homomorphism $G : \mathcal{A} \to \mathcal{A}$ satisfying* (3.22) *and a unique hom-derivation $F : \mathcal{A} \to \mathcal{A}$ satisfying* (2.9) *and* (3.21).

Proof. Let $\lambda = 1$ in (2.3) and (2.4). By Theorem 2.4 in [20], there are unique additive mappings $F, G : \mathcal{A} \to \mathcal{A}$ satisfying (3.21) and (3.22), respectively, which are given by

$$G(a) = \lim_{k\to\infty} \frac{1}{2^k} g\left(2^k a\right)$$

and

$$F(a) = \lim_{k\to\infty} \frac{1}{2^k} f\left(2^k a\right)$$

for all $a \in \mathcal{A}$. The rest of the proof is similar to the proof of Theorem 3.4. □

Corollary 3.4. *Let $r < 1$ and θ be positive real numbers, and let $f, g : \mathcal{A} \to \mathcal{A}$ be mappings satisfying $f(0) = g(0) = 0$ and* (2.11)-(2.14). *Then, there exist a unique homomorphism G : $\mathcal{A} \to \mathcal{A}$ satisfying* (2.19) *and a unique hom-derivation $F : \mathcal{A} \to \mathcal{A}$ satisfying* (2.9) *and* (2.18).

Proof. The proof follows from Theorem 3.5 by taking $\varphi(a, b) = \theta\left(\|a\|^r + \|b\|^r\right)$ for all $a, b \in \mathcal{A}$. Choosing $L = 2^{r-1}$, we obtain the desired result. □

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