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Existence and stability results for multi-term fractional delay differential equations equipped with nonlocal multi-point and multi-strip boundary conditions

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ABSTRACT. In this paper, we introduce and investigate a new class of nonlocal boundary value problems containing multi-term delay fractional differential equations and nonlocal multi-point and multi-strip boundary conditions. The existence results for the given problem are obtained applying the nonlinear alternative of Leray-Schauder type and Krasnoselkii's fixed point theorem, while the uniqueness of solutions is established with the aid of the Banach contraction mapping principle. We also study the stability criteria such as, generalized Ulam-Hyers, Ulam-Hyers-Rassias, and generalized Ulam-Hyers-Rassias stability, for the problem at hand. Examples are given to demonstrate the application of the obtained results. Some interesting observation are also presented.

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