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Finite-Time H_∞ Control for Conformable Fractional-Order Nonlinear Systems with Time Delays

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ABSTRACT. This paper studies finite-time H_∞ control for conformable fractional-order nonlinear systems with time delays. Unlike Caputo derivatives, conformable derivatives lack memory effects, affecting stability. Existing work on H_∞ control mainly covers integer-order and Caputo fractional systems, with little focus on conformable systems with delays. Using Lyapunov methods and the conformable Laplace transform, we derive linear matrix inequalities (LMIs) that ensure finite-time stability and robust H_∞ performance. A state-feedback control law is designed to improve system robustness. Numerical simulations validate the approach against disturbances and delays.

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