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Subgradient-extragradient algorithms with double inertial terms for solving image restoration problems

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ABSTRACT. In this study, we develop a class of subgradient-extragradient schemes enhanced with dual inertial mechanisms, aimed at resolving variational inequalities and fixed point formulations, particularly those arising in image reconstruction. By incorporating two inertial correction terms, the proposed algorithms aim to accelerate convergence while maintaining stability and robustness. We rigorously establish strong convergence results under suitable assumptions, extending existing theoretical frameworks to accommodate the added inertial dynamics. Numerical experiments are conducted on optimal control problems and image restoration tasks to demonstrate the practical efficiency and effectiveness of the proposed methods.

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