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Some De Morgan algebras associated with a double fuzzy topological space

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ABSTRACT. This paper identifies a De Morgan algebra associated with the families of (r, s)-regular fuzzy open sets and (r, s)-regular fuzzy closed sets in a double fuzzy topological space. The situation under which this De Morgan algebra becomes a Boolean algebra is characterized. Certain other properties of this algebra are also investigated.

1. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Attanassov [1] in 1983 and subsequently D. Çocker [3] studied intuitionistic fuzzy topological space and established several of its properties in 1997. Further, Gurcay H, et al. [6] extended the concept of fuzzy regular open sets and fuzzy regular closed sets to intuitionistic fuzzy topological spaces.

In 2001, S. Bayhan and D. Çoker [2] studied separation axioms in intuitionistic fuzzy topological spaces. The concept of intuitionistic gradation of openness was introduced by Mondal and Samanta [11] in 2002. Further, Hausdorffness, normality and regularity of intuitionistic fuzzy topological spaces were studied by Lupiáñez [9] in 2004.

Moreover, in 2005 Kim and Ramadan investigated connectedness and compactness in intuitionistic fuzzy topological spaces in [7] and [12] respectively. The concepts of r-closure and r-interior were introduced by Min et al. in [10].

Conforming to the view of J. G. García and S. E. Rodabaugh [5] that Intuitionistic Fuzzy Sets by definition cannot be Intuitionistic Mathematics, we use the term "double fuzzy topological spaces" instead of "intuitionistic fuzzy topological spaces" and some recent studies on double fuzzy topological spaces can be seen in [13], [14] and [15].

In this paper, the authors study various properties of $\mathcal{D}_{r,s} = \mathcal{RO}_{r,s} \cap \mathcal{RC}_{r,s}$ where $\mathcal{RO}_{r,s}$ is the family of (r, s)-regular fuzzy open sets and $\mathcal{RC}_{r,s}$ is the family of (r, s)-regular fuzzy closed sets in a double fuzzy topological space. As a result, $\mathcal{D}_{r,s}$ is found to be a De Morgan algebra. Besides, the situation under which $\mathcal{D}_{r,s}$ is a Boolean algebra is characterized. In general, $\mathcal{D}_{r,s}$ is neither atomic nor dual atomic and not a complete lattice.

2. PRELIMINARIES

Throughout the paper, X denotes a non-empty set, $I = [0, 1]$, $I_0 = (0, 1]$, $I_1 = [0, 1)$, $I^X =$ the set of all fuzzy subsets of X. The constant fuzzy subset taking the value α is denoted by $\underline{\alpha}$. Also, the complement of a fuzzy set f is denoted by f^c and the characteristic function of a crisp set A is denoted by χ_A.

Definition 2.1. [11] Consider the pair (τ, τ^*) of functions from $I^X \to I$ such that

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(i) $\tau(f) + \tau^*(f) \leq 1, \forall f \in I^X$,
(ii) $\tau(\underline{0}) = \tau(\underline{1}) = 1, \tau^*(\underline{0}) = \tau^*(\underline{1}) = 0$,
(iii) $\tau(f_1 \wedge f_2) \geq \tau(f_1) \wedge \tau(f_2)$ and $\tau^*(f_1 \wedge f_2) \leq \tau^*(f_1) \vee \tau^*(f_2), f_i \in I^X, i = 1, 2$,
(iv) $\tau(\bigvee_{i\in\Delta} f_i) \geq \bigwedge_{i\in\Delta} \tau(f_i)$ and $\tau^*(\bigvee_{i\in\Delta} f_i) \leq \bigvee_{i\in\Delta} \tau^*(f_i), f_i \in I^X, i \in \Delta$

The pair (τ, τ^*) is called a double fuzzy topology on X. The triplet (X, τ, τ^*) is called a double fuzzy topological space.

Definition 2.2. [8] Let (X, τ, τ^*) be a double fuzzy topological space. For each $r \in I_0, s \in I_1, f \in I^X$ the operator $C_{\tau,\tau^*} : I^X \times I_0 \times I_1 \to I^X$ defined by

$$C_{\tau,\tau^*}(f, r, s) = \bigwedge\{g \in I^X | f \leq g, \tau(g^c) \geq r, \tau^*(g^c) \leq s\}$$

is called the double fuzzy closure operator on (X, τ, τ^*).

Definition 2.3. [8] Let (X, τ, τ^*) be a double fuzzy topological space. For each $r \in I_0, s \in I_1, f \in I^X$ the operator $I_{\tau,\tau^*} : I^X \times I_0 \times I_1 \to I^X$ defined by

$$I_{\tau,\tau^*}(f, r, s) = \bigvee\{g \in I^X | f \geq g, \tau(g) \geq r, \tau^*(g) \leq s\}$$

is called the double fuzzy interior operator on (X, τ, τ^*).

Definition 2.4. [4] A De Morgan algebra is a structure $A = (A, \vee, \wedge, 0, 1,')$ such that:
(i) $(A, \vee, \wedge, 0, 1)$ is a bounded distributive lattice,
(ii) $'$ is a De Morgan involution: $(a \wedge b)' = a' \vee b'$ and $(a')' = a$.

Definition 2.5. [4] A Boolean algebra is a structure $\langle B; \vee, \wedge,', 0, 1\rangle$ such that
(i). $\langle B; \vee, \wedge\rangle$ is a distributive lattice.
(ii). $a \vee 0 = a$ and $a \wedge 1 = a$ for all $a \in B$.
(iii). $a \vee a' = 1$ and $a \wedge a' = 0$ for all $a \in B$.

Definition 2.6. [4] A poset L is called a
(i) join-semilattice if $x \vee y \in L$ for all $x, y \in L$ and
(ii) meet-semilattice if $x \wedge y \in L$ for all $x, y \in L$.

L is called a lattice if it is both join-semilattice and meet-semilattice.

3. De Morgan Algebra associated with a DFTS

In 2005, Ramadan et.al. [12] introduced regular fuzzy open sets and regular fuzzy closed sets in a double fuzzy topological space as:

Definition 3.7. Let (X, τ, τ^*) be a double fuzzy topological space, $f \in I^X, r \in I_0$ and $s \in I_1$. Then f is called
(i) (r, s)-regular fuzzy open (or (r, s)-rfo) if $f = I_{\tau,\tau^*}\big(C_{\tau,\tau^*}(f, r, s), r, s\big)$.
(ii) (r, s)-regular fuzzy closed (or (r, s)-rfc) if $f = C_{\tau,\tau^*}\big(I_{\tau,\tau^*}(f, r, s), r, s\big)$.

Notation: [14] Let $I_0 \oplus I_1 = \{(r, s) : r \in I_0, s \in I_1$ with $r + s \leq 1\}$.
For a given double fuzzy topological space (X, τ, τ^*) and $(r, s) \in I_0 \oplus I_1$, set
$\mathcal{RO}_{r,s} = \{f \in I^X : f$ is a (r, s)-regular fuzzy open set$\}$
$\mathcal{RC}_{r,s} = \{f \in I^X : f$ is a (r, s)-regular fuzzy closed set$\}$ and
$\mathcal{D}_{r,s} = \mathcal{RO}_{r,s} \cap \mathcal{RC}_{r,s}$.

With respect to the above notations, a straightforward observation given in [12] takes the following form:

Theorem 3.1. Let (X, τ, τ^*) be a double fuzzy topological space and $(r, s) \in I_0 \oplus I_1$. Then,

(i) $f \in \mathcal{RO}_{r,s} \Rightarrow \tau(f) \geq r$ and $\tau^*(f) \leq s$ and
(ii) $f \in \mathcal{RC}_{r,s} \Rightarrow \tau(f^c) \geq r$ and $\tau^*(f^c) \leq s$.

Moreover, the relationship between the families $\mathcal{RO}_{r,s}$ and $\mathcal{RC}_{r,s}$ is given by

Theorem 3.2. [14] Let (X, τ, τ^*) be a double fuzzy topological space and $(r, s) \in I_0 \oplus I_1$. Then, $\mathcal{RO}_{r,s} = \{f^c : f \in \mathcal{RC}_{r,s}\}$.

Obviously, the intersection of the above two subfamilies of I^X is non-empty. Indeed,

Lemma 3.1. [14] Let (X, τ, τ^*) be a double fuzzy topological space and $(r, s) \in I_0 \oplus I_1$. Then, $\underline{0}, \underline{1} \in \mathcal{D}_{r,s}$.

It should be noted that $\mathcal{D}_{r,s}$ may reduce to $\{\underline{0}, \underline{1}\}$ as shown by the following example.

Example 3.1. Let $X = \{a, b\}$ and consider the fuzzy set

$$g(x) = \begin{cases} \frac{1}{3}, & \text{if } x = a \\ \frac{1}{2}, & \text{if } x = b \end{cases}.$$

Now, define a double fuzzy topology on X as follows:

$$\tau(f) = \begin{cases} 1, & \text{if } f \in \{\underline{0}, \underline{1}\} \\ \frac{2}{3}, & \text{if } f = g \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad \tau^*(f) = \begin{cases} 0, & \text{if } f \in \{\underline{0}, \underline{1}\} \\ \frac{1}{4}, & \text{if } f = g \\ 1, & \text{otherwise.} \end{cases}$$

Then, for $r = \frac{3}{5}$ and $s = \frac{2}{5}$, $I_{\tau,\tau^*}\left(C_{\tau,\tau^*}\left(g, r, s\right), r, s\right) = I_{\tau,\tau^*}\left(g^c, r, s\right) = g$.
i.e., $g \in \mathcal{RO}_{r,s}$. Moreover, by Theorem 3.1 $\mathcal{RO}_{r,s} = \{\underline{0}, \underline{1}, g\}$ since there does not exist any $f \in I^X \setminus \{\underline{0}, \underline{1}, g\}$ such that $\tau(f) \geq r$ and $\tau^*(f) \leq s$.
Again, $C_{\tau,\tau^*}\left(I_{\tau,\tau^*}\left(g^c, r, s\right), r, s\right) = C_{\tau,\tau^*}\left(g, r, s\right) = g^c$. Hence, $g^c \in \mathcal{RC}_{r,s}$. Further, there does not exist any $f \in I^X \setminus \{\underline{0}, \underline{1}, g^c\}$ such that $\tau(f^c) \geq r$ and $\tau^*(f^c) \leq s$. Hence by Theorem 3.1, $\mathcal{RC}_{r,s} = \{\underline{0}, \underline{1}, g^c\}$.
Thus, $\mathcal{D}_{r,s} = \{\underline{0}, \underline{1}\}$. Here it should be noted that $\mathcal{D}_{r,s} \neq I^X$.

Now, we have

Theorem 3.3. [14] $\mathcal{D}_{r,s} = \mathcal{RO}_{r,s} \cap \mathcal{RC}_{r,s}$ is a bounded distributive lattice.

Moreover,

Theorem 3.4. $\mathcal{D}_{r,s}$ is a De Morgan algebra.

Proof. By Theorem 3.3, $\mathcal{D}_{r,s}$ is a bounded distributive lattice.
Now, $f \in \mathcal{D}_{r,s} \Rightarrow f \in \mathcal{RO}_{r,s} \cap \mathcal{RC}_{r,s}$. Then by Theorem 3.1, $\tau(f) \geq r$, $\tau^*(f) \leq s$, $\tau(f^c) \geq r$ and $\tau^*(f^c) \leq s$.
Therefore, $I_{\tau,\tau^*}\big(f^c, r, s\big) = f^c$ and $C_{\tau,\tau^*}\big(f^c, r, s\big) = f^c$. Consequently,

$$I_{\tau,\tau^*}(C_{\tau,\tau^*}(f^c, r, s), r, s) = I_{\tau,\tau^*}\big(f^c, r, s\big) = f^c.$$

i.e., $f^c \in \mathcal{RO}_{r,s}$. Similarly, $f^c \in \mathcal{RC}_{r,s}$. Hence, $f^c \in \mathcal{D}_{r,s}$.
i.e., $\mathcal{D}_{r,s}$ has a De Morgan involution viz., the complementation in I^X.
Hence, $\mathcal{D}_{r,s}$ is a De Morgan algebra. □

Remark 3.1. In general, $\mathcal{D}_{r,s}$ is not a Boolean algebra. For example, let $X = I$ and consider the fuzzy sets

$$f_1(x) = \begin{cases} \frac{1}{3}, & \text{if } x = \frac{1}{2} \\ \frac{3}{5}, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}, \quad f_2(x) = \begin{cases} \frac{3}{5}, & \text{if } x = \frac{1}{2} \\ \frac{7}{20}, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}, \quad f_3(x) = \begin{cases} \frac{2}{3}, & \text{if } x = \frac{1}{2} \\ \frac{3}{5}, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases},$$

$f_4 = f_1 \wedge f_2$, $f_5 = f_1 \vee f_2$. Then, define a double fuzzy topology (τ, τ^*) on X as follows:

$$\tau(f) = \begin{cases} 1, & \text{if } f \in \{\underline{0}, \underline{1}\} \\ \frac{2}{5}, & \text{if } f \in \{f_1, f_2, f_5\} \\ \frac{9}{20}, & \text{if } f \in \{f_4, f_3\} \\ \lambda, & \text{if } f = \left(\frac{1}{2}\right)_\lambda \text{ or } f = \left(\frac{1}{2}\right)^c_\lambda \text{ with } \lambda \in \left[\frac{1}{10}, \frac{1}{3}\right) \\ \frac{7}{20}, & \text{if } f = \left(\frac{1}{2}\right)_{\frac{1}{3}} \text{ or } f = \left(\frac{1}{2}\right)_{\frac{2}{5}} \\ 0, & \text{otherwise} \end{cases}$$

and

$$\tau^*(f) = \begin{cases} 0, & \text{if } f \in \{\underline{0}, \underline{1}\} \\ \frac{9}{20}, & \text{if } f \in \{f_1, f_2, f_5\} \\ \frac{1}{2}, & \text{if } f \in \{f_4, f_3\} \\ \lambda, & \text{if } f = \left(\frac{1}{2}\right)_\lambda \text{ or } f = \left(\frac{1}{2}\right)^c_\lambda \text{ with } \lambda \in \left[\frac{1}{10}, \frac{1}{3}\right) \\ \frac{7}{20}, & \text{if } f = \left(\frac{1}{2}\right)_{\frac{1}{3}} \text{ or } f = \left(\frac{1}{2}\right)_{\frac{2}{5}} \\ 1, & \text{otherwise.} \end{cases}$$

Let $(r, s) = (\frac{1}{10}, \frac{1}{3})$. For any $\lambda \in [\frac{1}{10}, \frac{1}{3})$,

$$\begin{aligned} C_{\tau,\tau^*}\left(I_{\tau,\tau^*}\left(\left(\frac{1}{2}\right)_\lambda, r, s\right), r, s\right) &= C_{\tau,\tau^*}\left(\left(\frac{1}{2}\right)_\lambda, r, s\right) \\ &= \left(\frac{1}{2}\right)_\lambda. \end{aligned}$$

i.e., $\left(\frac{1}{2}\right)_\lambda \in \mathcal{RC}_{r,s}$ for all $\lambda \in [\frac{1}{10}, \frac{1}{3})$.

Again, for any $\lambda \in [\frac{1}{10}, \frac{1}{3})$,

$$\begin{aligned} C_{\tau,\tau^*}\left(I_{\tau,\tau^*}\left(\left(\frac{1}{2}\right)^c_\lambda, r, s\right), r, s\right) &= C_{\tau,\tau^*}\left(\left(\frac{1}{2}\right)^c_\lambda, r, s\right) \\ &= \left(\frac{1}{2}\right)^c_\lambda. \end{aligned}$$

i.e., $\left(\frac{1}{2}\right)^c_\lambda \in \mathcal{RC}_{r,s}$ for all $\lambda \in [\frac{1}{10}, \frac{1}{3})$.

Hence, $\{ \left(\frac{1}{2}\right)_\lambda, \left(\frac{1}{2}\right)^c_\lambda : \lambda \in [\frac{1}{10}, \frac{1}{3})\} \subseteq \mathcal{RC}_{r,s}$.

Now, by Theorem 3.1, $f \in \mathcal{RC}_{r,s} \Rightarrow \tau(f^c) \geq r$ and $\tau^*(f^c) \leq s$. Since, there does not exist any $f \in I^X \setminus \left(\{ \left(\frac{1}{2}\right)_\lambda, \left(\frac{1}{2}\right)^c_\lambda : \lambda \in [\frac{1}{10}, \frac{1}{3})\} \cup \{\underline{0}, \underline{1}\}\right)$ such that $\tau(f^c) \geq r$ and $\tau^*(f^c) \leq s$, $\mathcal{RC}_{r,s} = \{ \left(\frac{1}{2}\right)_\lambda, \left(\frac{1}{2}\right)^c_\lambda : \lambda \in \left[\frac{1}{10}, \frac{1}{3}\right)\} \cup \{\underline{0}, \underline{1}\}$.

Therefore, by Theorem 3.2 $\mathcal{RO}_{r,s} = \{ \left(\frac{1}{2}\right)_\lambda, \left(\frac{1}{2}\right)^c_\lambda : \lambda \in \left[\frac{1}{10}, \frac{1}{3}\right)\} \cup \{\underline{0}, \underline{1}\}$.
Consequently,

$$\mathcal{D}_{r,s} = \left\{ \left(\frac{1}{2}\right)_\lambda, \left(\frac{1}{2}\right)^c_\lambda : \lambda \in \left[\frac{1}{10}, \frac{1}{3}\right) \right\} \cup \{\underline{0}, \underline{1}\},$$

which is not a Boolean algebra.

The following theorem gives a necessary and sufficient condition for $\mathcal{D}_{r,s}$ to be a Boolean algebra.

Theorem 3.5. Let (X, τ, τ^*) be a double fuzzy topological space. Then $\mathcal{D}_{r,s}$ is a Boolean algebra if and only if $\mathcal{D}_{r,s} \subseteq \{\chi_A : A \subseteq X\}$.

Proof. Suppose $\mathcal{D}_{r,s}$ is a Boolean algebra with complementation $'$ and let $f \in \mathcal{D}_{r,s}$. If there exists $x \in X$ such that $0 < f(x) < 1$ then for every $g \in I^X$, either $(f \vee g) \neq \underline{1}$ or $(f \wedge g) \neq \underline{0}$ which shows that f' does not exist. Consequently, $\mathcal{D}_{r,s} \subseteq \{\chi_A : A \subseteq X\}$.

Conversely, suppose $\mathcal{D}_{r,s} \subseteq \{\chi_A : A \subseteq X\}$. By Theorem 3.4, $\mathcal{D}_{r,s}$ is a De Morgan algebra with complementation as the De Morgan involution. Also, $\underline{0}, \underline{1} \in \mathcal{D}_{r,s}$ with $f \wedge \underline{1} = f$ and $f \vee \underline{0} = f$, $\forall f \in \mathcal{D}_{r,s}$. Further, since $\mathcal{D}_{r,s} \subseteq \{\chi_A : A \subseteq X\}$ we have $f \vee f^c = \underline{1}$ and $f \wedge f^c = \underline{0}$, for all $f \in \mathcal{D}_{r,s}$. □

In general, $\mathcal{D}_{r,s}$ has no atoms and dual atoms as shown below:

Example 3.2. Let $X = I$ and $\mathcal{F} = \left\{\underline{\alpha}, \underline{\alpha}^c : 0 < \alpha \leq \frac{1}{4}\right\}$. Then, consider the double fuzzy topology (τ, τ^*) on X defined by

$$\tau(f) = \begin{cases} 1, & \text{if } f \in \{\underline{0}, \underline{1}\} \\ \frac{1}{4}, & \text{if } f \in \mathcal{F} \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad \tau^*(f) = \begin{cases} 0, & \text{if } f \in \{\underline{0}, \underline{1}\} \\ \frac{1}{4}, & \text{if } f \in \mathcal{F} \\ 1, & \text{otherwise.} \end{cases}$$

Let $r = \frac{1}{4}$ and $s = \frac{1}{2}$. Then, for any $f \in \mathcal{F}$, $I_{\tau,\tau^*}(f, r, s) = f$ and $C_{\tau,\tau^*}(f, r, s) = f$. Therefore, $C_{\tau,\tau^*}(I_{\tau,\tau^*}(f, r, s), r, s) = f$ and $I_{\tau,\tau^*}(C_{\tau,\tau^*}(f, r, s), r, s) = f$.

Hence, $\mathcal{RO}_{\tau,r,s} = \mathcal{F} \cup \{\underline{0}, \underline{1}\}$ and $\mathcal{RC}_{\tau,r,s} = \mathcal{F} \cup \{\underline{0}, \underline{1}\}$.

Thus, $\mathcal{D}_{r,s} = \mathcal{F} \cup \{\underline{0}, \underline{1}\}$ and it follows that no element of $\mathcal{F}$ can be an atom or dual atom of $\mathcal{D}_{r,s}$.

Remark 3.2. In general, $\mathcal{D}_{r,s}$ is not a fuzzy topology and hence not a complete lattice. For example, consider the double fuzzy topology (τ, τ^*) on X defined in Remark 3.1. Then, we have, $\mathcal{D}_{\frac{1}{10},\frac{1}{3}} = \left\{\left(\frac{1}{2}\right)_\lambda, \left(\frac{1}{2}\right)^c_\lambda : \lambda \in [\frac{1}{10}, \frac{1}{3})\right\} \cup \{\underline{0}, \underline{1}\}$. Clearly, $\bigvee_{\lambda \in [\frac{1}{10}, \frac{1}{3})} (\frac{1}{2})_\lambda = (\frac{1}{2})_{\frac{1}{3}} \notin \mathcal{D}_{\frac{1}{10},\frac{1}{3}}$ and $\bigwedge_{\lambda \in [\frac{1}{10}, \frac{1}{3})} (\frac{1}{2})^c_\lambda = (\frac{1}{2})^c_{\frac{1}{3}} \notin \mathcal{D}_{\frac{1}{10},\frac{1}{3}}$, which shows that $\mathcal{D}_{\frac{1}{10},\frac{1}{3}}$ is not a fuzzy topology and not a complete lattice.

4. Conclusion

A De Morgan algebra $\mathcal{D}_{r,s}$ associated with a double fuzzy topological space, which is the intersection of the family $\mathcal{RO}_{r,s}$ of (r, s)-regular fuzzy open sets and the family $\mathcal{RC}_{r,s}$ of (r, s)-regular fuzzy closed sets, is identified. Further, a situation under which $\mathcal{D}_{r,s}$ becomes a Boolean algebra is characterized. Moreover, it is shown that $\mathcal{D}_{r,s}$ is not a complete lattice and does not admit atoms and dual atoms, in general.

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