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More on δ- and θ-modifications

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ABSTRACT. Using δ-and θ-modifications of bigeneralized topologies we introduce $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity and related maps between two BIGTS's. We characterize these maps using the concepts of mixed generalized open sets: δ_{ν_1,ν_2}-open sets, θ_{ν_1,ν_2}-open sets.

1. INTRODUCTION

Ákos Császár [3], introduced and study the concept of generalized topological spaces and he also introduces two kinds of generalized continuity in [3] and again from the studies of Ákos Császár (see [3],[5]) we have learned that the theory of δ- and θ-modifications of topological spaces [13] can be taken to generalized topology. As a continuation of this study in [7], Csaszar and Makai introduced δ_{ν_1,ν_2}-open sets and θ_{ν_1,ν_2}-open sets defined by two given generalized topologies ν_1, ν_2 on a set X $(\neq \emptyset)$. They introduced the notion of $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity, and they showed that every (ν_1, υ_1)-continuous and (ν_2, υ_2)-continuous map is $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuous. In [10] W. K. Min, gave a characterization for $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity and he introduced the concept of $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity between bigeneralized topological spaces and studied the relations between $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity and $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity. W. K. Min in ([9],[11]) studied other related concepts of continuity. In this paper, we aim to extend the implication table in Remark 3. 5. of [10], in fact we look for possible relatives for the maps in this table, for doing this in the first step we will introduce *weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$ continuity, $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity, $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity, on bigeneralized topological spaces and after that we will study the implications between $(\theta_{\nu_1,\nu_2},\ \theta_{\upsilon_1,\upsilon_2})$-continuity and $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$ -continuity with these three maps. W.K. Min ([10]) pointed out that there is no relation between $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity and $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity. But $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity and the other maps is slightly relates these two variants of maps, for example both $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity and $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity implies $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuity.

2. PRELIMINARIES

Consider a set X $(\neq \emptyset)$, a subfamily $\nu \subset \exp X$, where $\exp X$ is the power set of X, is called a generalized topology [3] (briefly, GT) on X if $\emptyset \in \nu$ and ν is closed under union. A set X $(\neq \emptyset)$ with a GT ν on X is called a generalized topological space (briefly, GTS) and is denoted by (X, ν). A GTS (X, ν) is called strong if $X \in \nu$. For a GTS (X, ν), the elements of ν are called ν-open sets and the complements of ν-open sets are called ν-closed sets [3]. The intersection of all ν-closed sets containing a subset S of X is denoted by $c_\nu(S)$, and the union of all ν-open sets contained in S is denoted by $i_\nu(S)$, (see [4], [5]). Note that

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$c_\nu(X\diagdown S) = X\diagdown i_\nu(S)$. It is known that $i_\nu(S)$ and $c_\nu(S)$ are idempotent and monotonic [5]. Let ν and υ be GT's on X and Y, respectively, then a map $f : (X, \nu) \to (Y, \upsilon)$ is called (ν, υ)-continuous (briefly (ν, υ)-c.) map [3] (or generalized continuous) if $G \in \upsilon$ implies that inverse of G under f is in ν, that is $f^{\leftarrow}(G) \in \nu$. Let ν_1,ν_2 be two GT's on a set X $(\neq \emptyset)$. Then $(X; \nu_1, \nu_2)$ called a bigeneralized topological space [10] (briefly BIGTS). Let $(X; \nu_1, \nu_2)$ be a BIGTS and $S \subseteq X$. S is said to be r_{ν_1,ν_2}-open (respectively, r_{ν_1,ν_2}-closed) [6] if $S = i_{\nu_1}(c_{\nu_2}(S))$ (respectively, $S = c_{\nu_1}(i_{\nu_2}(S))$).

Definition 2.1. [6] Császár defined θ_{ν_1,ν_2}, $\delta_{\nu_1,\nu_2} \subseteq 2^X$ by
(1) $S \in \theta_{\nu_1,\nu_2}$ iff for each $x \in S$, there exists an $W \in \nu_1$ such that $x \in W \subseteq c_{\nu_2}(W) \subseteq S$;
(2) $S \in \delta_{\nu_1,\nu_2}$ iff $S \subseteq X$ and, if $x \in S$, then there is a ν_2-closed set F such that $x \in i_{\nu_1}(F) \subseteq S$.

An element S in δ_{ν_1,ν_2} (respectively, θ_{ν_1,ν_2}) is called a δ_{ν_1,ν_2}-open (respectively, θ_{ν_1,ν_2}-open) set [6]. S is called a δ_{ν_1,ν_2}-closed (respectively, θ_{ν_1,ν_2}-closed) set if the complement of S is δ_{ν_1,ν_2} -open (respectively, θ_{ν_1,ν_2}-open) [6].
Notations defined as follows:
$c_{\theta_{\nu_1,\nu_2}}(S) = \cap\{F \subseteq X : S \subseteq F$ for θ_{ν_1,ν_2}-closed set F in $X\}$ [9] ;
$i_{\theta_{\nu_1,\nu_2}}(S) = \cup\{V \subseteq X : V \subseteq S$ for $V \in \theta_{\nu_1,\nu_2}\}$ [9];
$i_{\delta_{\nu_1,\nu_2}}(S) = \cup\{V \subseteq X : V \subseteq S$ for $V \in \delta_{\nu_1,\nu_2}\} = \cup\{V \subseteq X : V \subseteq S$ for r_{ν_1,ν_2}-open set V in $X\}$;
$\gamma_{\theta_{\nu_1,\nu_2}}(S) = \{x \in X : c_{\nu_2}(W) \cap S \neq \emptyset$ for every $W \in \nu_1$ containing $x\}$ [7].

Lemma 2.1. [6] *Let ν_1 and ν_2 be two GT on a set X $(\neq \emptyset)$ and $S \subseteq X$. The following are valid:*
(1) θ_{ν_1,ν_2} and δ_{ν_1,ν_2} are GT's on X and $\theta_{\nu_1,\nu_2} \subseteq \delta_{\nu_1,\nu_2} \subseteq \nu_1$.
(2) The nonempty elements of δ_{ν_1,ν_2} coincides with the unions of r_{ν_1,ν_2} -open sets.
(3) $x \in c_{\delta_{\nu_1,\nu_2}}(S)$ iff $S \cap V \neq \emptyset$ for every r_{ν_1,ν_2} -open set V containing x .
(4) If F is ν_2-closed, then $i_{\nu_1}(F)$ is r_{ν_1,ν_2}-open.

Lemma 2.2. *Let ν_1 and ν_2 be two GT on a set X $(\neq \emptyset)$ and $S \subseteq X$. Then the following hold:*
(1) $S \subseteq \gamma_{\theta_{\nu_1,\nu_2}}(S) \subseteq c_{\theta_{\nu_1,\nu_2}}(S)$ [7].
(2) S is θ_{ν_1,ν_2}-closed iff $S = \gamma_{\theta_{\nu_1,\nu_2}}(S)$ [7].
(3) $x \in i_{\theta_{\nu_1,\nu_2}}(S)$ iff there exists a ν_1-open set W containing x such that $x \in W \subseteq c_{\nu_2}(W) \subseteq S$ [10].
(4) If S is ν_2-open in X, then $\gamma_{\theta_{\nu_1,\nu_2}}(S) = c_{\nu_1}(S)$ [7].

Definition 2.2. Let $(X; \nu_1, \nu_2)$ and $(Y; \upsilon_1, \upsilon_2)$ be BIGTS's. A map $f : X \to Y$ is said to be;
(1) $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-*continuous* [6] *(briefly* $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c.) *if for every* $G \in \theta_{\upsilon_1,\upsilon_2}$, $f^{\leftarrow}(G) \in \theta_{\nu_1,\nu_2}$,
(2) $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-*continuous* [10] *(briefly* $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-c.*) if for every* $G \in \delta_{\upsilon_1,\upsilon_2}$, $f^{\leftarrow}(G) \in \delta_{\nu_1,\nu_2}$.

Definition 2.3. Let (X, ν) be a GT and $(Y; \upsilon_1, \upsilon_2)$ be a BIGTS. A map $f : (X, \nu) \to (Y; \upsilon_1, \upsilon_2)$ is said to be faintly $(\nu : \upsilon_1\upsilon_2)$-*continuous* [11] *(briefly* faintly $(\nu : \upsilon_1\upsilon_2)$-*c.*) *if* for every $V \in \theta_{\upsilon_1,\upsilon_2}$ we have $f^{\leftarrow}(V) \in \nu$.

Consider a bigeneralized topology $(X; \nu_1, \nu_2)$ the notation $\mathcal{O}_{\delta_{\nu_1,\nu_2}}(X, x)$ (respectively $\mathcal{O}_{\theta_{\nu_1,\nu_2}}(X, x)$, $\mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$, $\mathcal{O}_{\nu_i}(X, x)$ $(i \in \{1, 2\})$) will be used for the family of δ_{ν_1,ν_2}-open sets (respectively θ_{ν_1,ν_2}-open sets, r_{ν_1,ν_2}-open sets, ν_i-open sets $(i \in \{1, 2\})$) containing a point $x \in X$.

3. $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-CONTINUITY AND RELATED MAPS

Definition 3.4. Let $(X;\nu_1,\nu_2)$ and $(Y;\upsilon_1,\upsilon_2)$ be BIGTS's. A map $f : X \to Y$ is said to be $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuous *(briefly* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c.*)* if for every $G \in \theta_{\upsilon_1,\upsilon_2}$, $f^{\leftarrow}(G) \in \delta_{\nu_1,\nu_2}$.

Definition 3.5. Let $(X;\nu_1,\nu_2)$ and $(Y;\upsilon_1,\upsilon_2)$ be BIGTS's. A map $f : X \to Y$ is said to be $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuous *(briefly* $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-c.*)* if for every $G \in \delta_{\upsilon_1,\upsilon_2}$, $f^{\leftarrow}(G) \in \theta_{\nu_1,\nu_2}$.

$(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$ -continuity is related with both of the concepts of $\theta\delta$-continuity due to Santoro [12] and ij-weakly θ-continuity due to Khedr and Al-Shibani [8]. The following definition is a generalization of the concept of ij-weakly θ-continuity due to Khedr and Al-Shibani [8] to the bigeneralized topologies. Note that ij-weakly θ-continuity is a generalization of the concept of weakly θ-continuity due to Cammaroto and Noiri [2].

Definition 3.6. Let $(X;\nu_1,\nu_2)$ and $(Y;\upsilon_1,\upsilon_2)$ be BiGTS's. A map $f : X \to Y$ is said to be *weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$*-continuous (shortly weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$-c.*) if for each* $x \in X$ *and each* $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$*, there exists a* $U \in \mathcal{O}_{\nu_1}(X, x)$ such that $f(i_{\nu_1}(c_{\nu_2}(U))) \subseteq c_{\upsilon_2}(V)$.

Then we need working examples, especially for the weak $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$*-continuity, but* we will provide necessary examples in the last section.

Theorem 3.1. *Let* $(X;\nu_1,\nu_2)$ *and* $(Y;\upsilon_1,\upsilon_2)$ *be BIGTS's, for a map* $f : X \to Y$*, the following are equivalent:*
(1) f *is weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$*-c.*
(2) For each $x \in X$ *and each* $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$ *, there exists a* $W \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ *such that* $f(W) \subseteq c_{\upsilon_2}(V)$*.*
(3) For each $x \in X$ *and each* $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$*, there exists an* $U \in \mathcal{O}_{\delta_{\nu_1,\nu_2}}(X, x)$ *such that* $f(U)) \subseteq c_{\upsilon_2}(V)$*.*

Proof. $(1) \Rightarrow (2)$: Let $x \in X$ and $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$, by (1) there exists a $U \in \mathcal{O}_{\nu_1}(X, x)$ *such that* $f(i_{\nu_1}(c_{\nu_2}(U))) \subseteq c_{\upsilon_2}(V)$. Since $W = i_{\nu_1}(c_{\nu_2}(U))$ is r_{ν_1,ν_2}-open set we have $f(W) \subseteq c_{\upsilon_2}(V)$.
$(2) \Rightarrow (3)$: Let $x \in X$ and $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$ then (2) implies that there exists a $W \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ such that $f(W) \subseteq c_{\upsilon_2}(V)$. Since every r_{ν_1,ν_2}-open set is δ_{ν_1,ν_2}-open set (3) is true.
$(3) \Rightarrow (1)$: Let $x \in X$ and $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$. From (3) there exists $U \in \mathcal{O}_{\delta_{\nu_1,\nu_2}}(X, x)$ such that $f(U) \subseteq c_{\upsilon_2}(V)$. Since U is δ_{ν_1,ν_2}*-open set*, there is a ν_2-closed set F such that $x \in i_{\nu_1}(F) \subseteq U$, take $W = i_{\nu_1}(F)$, then $W \in \nu_1$ and since $i_{\nu_1}(F)$ is r_{ν_1,ν_2}-open, it is true that $W = i_{\nu_1}(c_{\nu_2}(W)) = i_{\nu_1}(F)$ hence we have $f(i_{\nu_1}(c_{\nu_2}(W))) \subseteq f(U)) \subseteq c_{\upsilon_2}(V)$. □

Theorem 3.2. *Let* $(X;\nu_1,\nu_2)$ *and* $(Y;\upsilon_1,\upsilon_2)$ *be BIGTS's, for a map* $f : X \to Y$ *the following are equivalent:*
(1) f *is weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$*-c.*
(2) $f(c_{\delta_{\nu_1,\nu_2}}(A)) \subseteq \gamma_{\theta_{\upsilon_1,\upsilon_2}}(f(A))$ *for every* $A \subseteq X$*.*
(3) $c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}\left(\gamma_{\theta_{\upsilon_1,\upsilon_2}}(B)\right)$ *for every* $B \subseteq Y$*.*
(4) $c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(V)) \subseteq f^{\leftarrow}(c_{\nu_1}(V))$ *for every* υ_2*-open subset* V *of* Y*.*

Proof. $(1) \Rightarrow (2)$: For $A \subseteq X$, let $x \in c_{\delta_{\nu_1,\nu_2}}(A)$ and $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$. Then since f is *weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$-c., there exists a $U \in \mathcal{O}_{\nu_1}(X, x)$ *such that* $f(i_{\nu_1}(c_{\nu_2}(U))) \subseteq c_{\upsilon_2}(V)$. Since $x \in c_{\delta_{\nu_1,\nu_2}}(A)$ and $i_{\nu_1}(c_{\nu_2}(U))$ is r_{ν_1,ν_2}-open set in X, we have $A \cap i_{\nu_1}(c_{\nu_2}(U)) \neq \emptyset$. So $\emptyset \neq f(A) \cap f(i_{\nu_1}(c_{\nu_2}(U))) \subseteq c_{\upsilon_2}(V) \cap f(A)$. Then we have $f(x) \in \gamma_{\theta_{\upsilon_1,\upsilon_2}}(f(A))$.
$(2) \Rightarrow (3)$: Taking $A = f^{\leftarrow}(B)$ in (2) $f(c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(B))) \subseteq \gamma_{\theta_{\upsilon_1,\upsilon_2}}(f(f^{\leftarrow}(B))) \subseteq \gamma_{\theta_{\upsilon_1,\upsilon_2}}(B)$.

Then we have $c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}\left(f(c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(B))\right) \subseteq f^{\leftarrow}\left(\gamma_{\theta_{\upsilon_1,\upsilon_2}}(B)\right)$.
(3) $\Rightarrow$ (4) : Let $V \in \upsilon_2$. Then by Lemma 2.2*(4)* , $\gamma_{\theta_{\nu_1,\nu_2}}(V) = c_{\nu_1}(V)$ and by (3) *we have* $c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(V)) \subseteq f^{\leftarrow}(c_{\nu_1}(V))$.
(4) $\Rightarrow$ (1) : Let $x \in X$ and $V \in \mathcal{O}_{\upsilon_1}(Y, f(x))$. Since $V = i_{\upsilon_1}(V) \subseteq i_{\upsilon_1}(c_{\upsilon_2}(V))$ and $Y \diagdown c_{\upsilon_2}(V) \in \upsilon_2$ by (4) we have $x \in f^{\leftarrow}(V) \subseteq f^{\leftarrow}(i_{\upsilon_1}(c_{\upsilon_2}(V))) = X \diagdown f^{\leftarrow}(c_{\upsilon_1}(Y \diagdown c_{\upsilon_2}(V)))$ $\subseteq X \diagdown c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(Y \diagdown c_{\upsilon_2}(V))) = i_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(c_{\upsilon_2}(V)))$. Then $x \in i_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(c_{\upsilon_2}(V)))$, so that there exists a $U \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ such that $U \subseteq f^{\leftarrow}(c_{\upsilon_2}(V))$, hence f is *weakly* $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$-c. □

Corollary 3.1. *Let $(X; \nu_1, \nu_2)$ and $(Y; \upsilon_1, \upsilon_2)$ be BIGTS's. If a map $f : (X; \nu_1, \nu_2) \to (Y; \upsilon_1, \upsilon_2)$ is weakly $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$-c. then the following are valid:*
(1) $f^{\leftarrow}(F)$ is δ_{ν_1,ν_2}-closed in X for each $\theta_{\upsilon_1,\upsilon_2}$-closed F in Y.
(2) $f^{\leftarrow}(V)$ is δ_{ν_1,ν_2}-open in X for each $\theta_{\upsilon_1,\upsilon_2}$-open V in Y.

Proof. Let F be $\theta_{\upsilon_1,\upsilon_2}$-closed in Y. By Theorem 3.2(3) and by Lemma 2.2(4) we have $c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(F)) \subseteq f^{\leftarrow}(F)$ and so that $f^{\leftarrow}(F)$ is δ_{ν_1,ν_2}-closed in X.
(2) is clear from (1). □

Theorem 3.3. *For BIGTS's $(X; \nu_1, \nu_2)$ and $(Y; \upsilon_1, \upsilon_2)$, if $f : X \to Y$ is $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuous, then for every $\theta_{\upsilon_1,\upsilon_2}$-closed M in $(Y; \upsilon_1, \upsilon_2)$, $f^{\leftarrow}(M)$ is δ_{ν_1,ν_2}-closed in $(X; \nu_1, \nu_2)$.*

Proof. . Assume M is $\theta_{\upsilon_1,\upsilon_2}$-closed in $(Y; \upsilon_1, \upsilon_2)$, since $f : X \to Y$ is $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-continuous we have $f^{\leftarrow}(Y \setminus M) = X \setminus f^{\leftarrow}(M)$ is δ_{ν_1,ν_2}-open in $(X; \nu_1, \nu_2)$. Hence $f^{\leftarrow}(M)$ is δ_{ν_1,ν_2}-closed in $(X; \nu_1, \nu_2)$. □

Theorem 3.4. *Let $(X; \nu_1, \nu_2)$ and $(Y; \upsilon_1, \upsilon_2)$ be BIGTS's, for a map $f : X \to Y$, the following are equivalent:*
(1) f is $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c.
(2) For each $x \in X$ and each $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, there exists a $U \in \mathcal{O}_{\delta_{\nu_1,\nu_2}}(X, x)$ such that $f(U) \subseteq V$.
(3) For each $x \in X$ and each $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, there exists a ν_2-closed set M containing x such that $f(i_{\nu_1}(M)) \subseteq V$.
(4) For each $x \in X$ and each $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, there exists a ν_1-open set C containing x such that $f(i_{\nu_1}(c_{\nu_2}(C))) \subseteq V$.
(5) For each $x \in X$ and each $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, there exists a $U \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ such that $f(U) \subseteq V$.

Proof. (1) $\Rightarrow$ (2) : Let $V \in \theta_{\upsilon_1,\upsilon_2}$ and $x \in f^{\leftarrow}(V)$. By (1) $f^{\leftarrow}(V) \in \delta_{\nu_1,\nu_2}$. Take $U = f^{\leftarrow}(V)$; then $U \in \mathcal{O}_{\delta_{\nu_1,\nu_2}}(X, x)$ and satisfies $f(U) \subseteq V$.
(2) $\Rightarrow$ (3) : Let $V \in \theta_{\upsilon_1,\upsilon_2}$ and $x \in f^{\leftarrow}(V)$. (2) implies that there exists a $W \in \mathcal{O}_{\delta_{\nu_1,\nu_2}}(X, x)$ such that $f(W) \subseteq V$. δ_{ν_1,ν_2}-openness of W implies that there exists a ν_2-closed set M containing x such that $x \in i_{\mu_1}(M) \subseteq W$, so we have $f(i_{\nu_1}(M)) \subseteq f(W) \subseteq V$. So there exists a ν_2-closed set M containing x *satisfying* $f(i_{\nu_1}(M)) \subseteq V$.
(3) $\Rightarrow$ (1) : Let $V \in \theta_{\upsilon_1,\upsilon_2}$. For each $x \in f^{\leftarrow}(V)$ there exists a ν_2-closed set M *satisfying* $x \in i_{\nu_1}(M)$ and $f(i_{\nu_1}(M)) \subseteq V$ by (3). So we have $x \in i_{\nu_1}(M) \subseteq f^{\leftarrow}(V)$, that is $f^{\leftarrow}(V)$ is δ_{ν_1,ν_2}-open.
(3) $\Rightarrow$ (4) : Let $V \in \theta_{\upsilon_1,\upsilon_2}$ and $x \in f^{\leftarrow}(V)$. By (3) there exists a ν_2*-closed set M satisfying* $x \in i_{\nu_1}(M)$ and $f(i_{\nu_1}(M)) \subseteq V$. Since $x \in i_{\nu_1}(M)$ there exists a $C \in \mathcal{O}_{\nu_1}(X, x)$ such that $x \in C \subseteq i_{\nu_1}(M) \subseteq M$ and from this containment we have $C \subseteq c_{\nu_2}(C) \subseteq c_{\nu_2}(i_{\nu_1}(M)) \subseteq c_{\nu_2}(M) = M$ and $x \in C \subseteq i_{\nu_1}(c_{\nu_2}(C)) \subseteq i_{\nu_1}(M)$ that is $f(i_{\nu_1}(c_{\nu_2}(C))) \subseteq V$.
(4) $\Rightarrow$ (5) : For each $x \in X$ and each $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, there exists a $B \in \mathcal{O}_{\nu_1}(X, x)$ such that $f(i_{\nu_1}(c_{\nu_2}(B))) \subseteq V$. Set $U = i_{\nu_1}(c_{\nu_2}(B))$ then we have $U \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ and

$f(U) \subseteq V$.
$(5) \Rightarrow (1)$: Let $V \in \theta_{\upsilon_1,\upsilon_2}$. Take $x \in f^{\leftarrow}(V)$ then $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, from (5) there exists a $U \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ such that $f(U) \subseteq V$ and then it is clear that $f^{\leftarrow}(V) \in \delta_{\nu_1,\nu_2}$ as a union of r_{ν_1,ν_2}-open sets. □

Theorem 3.5. *Let* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$ *be BIGTS's, for a map* $f : X \to Y$*, the following are equivalent:*
(1) f *is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(2) $f^{\leftarrow}\left(i_{\theta_{\upsilon_1,\upsilon_2}}(B)\right) \subseteq i_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(B))$ *for each* $B \subseteq Y$.

Proof. $(1) \Rightarrow (2)$: Let $x \in f^{\leftarrow}\left(i_{\theta_{\upsilon_1,\upsilon_2}}(B)\right)$, if $x \notin i_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(B))$ then $(X \setminus f^{\leftarrow}(B)) \cap V \neq \emptyset$ for every $V \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$. From the hypothesis $f^{\leftarrow}\left(i_{\theta_{\upsilon_1,\upsilon_2}}(B)\right) \in \delta_{\nu_1,\nu_2}$, hence there exists a $G \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$ such that $G \subseteq f^{\leftarrow}\left(i_{\theta_{\upsilon_1,\upsilon_2}}(B)\right) \subseteq f^{\leftarrow}(B)$, contradiction.
$(2) \Rightarrow (1)$: Let $V \in \theta_{\upsilon_1,\upsilon_2}$. Then $i_{\theta_{\upsilon_1,\upsilon_2}}(V) = V$ and by (2) $f^{\leftarrow}(V) \subseteq i_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(V))$, since reverse containment is always true, we have $f^{\leftarrow}(V) = i_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(V))$, that is $f^{\leftarrow}(V) \in \delta_{\nu_1,\nu_2}$. □

Theorem 3.6. *Let* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$ *be BIGTS's, for a map* $f : X \to Y$*, if the map* f *is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c. iff* $f(c_{\delta_{\nu_1,\nu_2}}(A)) \subseteq c_{\theta_{\upsilon_1,\upsilon_2}}(f(A))$ *for every* $A \subseteq X$.

Proof. Let $A \subseteq X$ and assume $y \in f(c_{\delta_{\nu_1,\nu_2}}(A))$, then there exists $x \in c_{\delta_{\nu_1,\nu_2}}(A)$ such that $y = f(x)$. Since $x \in c_{\delta_{\nu_1,\nu_2}}(A)$, $A \cap U \neq \emptyset$ for every $U \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$. By Theorem 3.4 for each $x \in X$ and each $V \in \mathcal{O}_{\theta_{\upsilon_1,\upsilon_2}}(Y, f(x))$, there exists a $U \in \mathcal{O}_{r_{\nu_1,\nu_2}}(X, x)$, such that $f(U) \subseteq V$, we have $V \cap f(A) \neq \emptyset$. Hence $f(x) \in c_{\theta_{\upsilon_1,\upsilon_2}}(f(A))$.
Conversely, let $B \in \theta_{\upsilon_1,\upsilon_2}$, then $Y \setminus B$ is $\theta_{\upsilon_1,\upsilon_2}$-closed set in Y, and since for arbitrary $C \subseteq Y$, $c_{\theta_{\upsilon_1,\upsilon_2}}(C)$ is $\theta_{\upsilon_1,\upsilon_2}$-closed set in Y, we have $c_{\theta_{\upsilon_1,\upsilon_2}}(Y \setminus B) = Y \setminus B$. Taking $A = f^{\leftarrow}(Y \setminus B)$ we get $f(c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(Y \setminus B))) \subseteq c_{\theta_{\upsilon_1,\upsilon_2}}(f(f^{\leftarrow}(Y \setminus B))) \subseteq c_{\theta_{\upsilon_1,\upsilon_2}}(Y \setminus B) = Y \setminus B$. So it is true that $c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(Y \setminus B)) \subseteq f^{\leftarrow}\left(f(c_{\delta_{\nu_1,\nu_2}}(f^{\leftarrow}(Y \setminus B)))\right) \subseteq f^{\leftarrow}(Y \setminus B)$ then $f^{\leftarrow}(B) \in \delta_{\nu_1,\nu_2}$. □

Remark 3.1. Considering Császàr's and Min's papers, *Bayhan, Kanıbir, Reilly* [1], *pointed out that* if $f : (X, \nu) \to (Y, \upsilon)$ is a weaker form of generalized continuity, then f can be made into a generalized continuous map by replacing ν or υ with some suitable generalized topologies mentioned above. Since δ_{ν_1,ν_2}-open sets and θ_{ν_1,ν_2}*-open sets again a GT on* X [7], we may state the following theorem*:*

Theorem 3.7. *For BIGTS's* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$*,* $f : X \to Y$ *is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c. iff* $f : (X, \delta_{\nu_1,\nu_2}) \to (Y, \theta_{\upsilon_1,\upsilon_2})$ *is generalized continuous.*

Proof. This is clear. □

Theorem 3.8. *For BIGTS's* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$*, if* $f : X \to Y$ *is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-continuous, then for every* δ_{ν_1,ν_2}*-closed* M *in* $(Y; \upsilon_1, \upsilon_2)$*,* $f^{\leftarrow}(M)$ *is* $\theta_{\upsilon_1,\upsilon_2}$*-closed in in* $(X; \nu_1, \nu_2)$.

Proof. Assume M is δ_{ν_1,ν_2}-closed in $(Y; \upsilon_1, \upsilon_2)$, since $f : X \to Y$ is $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-c. we have $f^{\leftarrow}(Y \setminus M) = X \setminus f^{\leftarrow}(M)$ is $\theta_{\upsilon_1,\upsilon_2}$-open in $(X; \nu_1, \nu_2)$. Hence $f^{\leftarrow}(M)$ is $\theta_{\upsilon_1,\upsilon_2}$-closed in $(X; \nu_1, \nu_2)$.

□

Theorem 3.9. *Let* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$ *be BIGTS's, for a map* $f : X \to Y$*, the following are equivalent:*
(1) f *is* $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$*-c.*
(2) For each $x \in X$ *and each* $V \in \mathcal{O}_{\delta_{\upsilon_1,\upsilon_2}}(Y, f(x))$*, there exists an* $U \in \mathcal{O}_{\theta_{\nu_1,\nu_2}}(X, x)$ *such that*

$f(U) \subseteq V$.
(3) For each $x \in X$ *and each* $V \in \mathcal{O}_{r_{\upsilon_1,\upsilon_2}}(Y, f(x))$*, there exists an* $U \in \mathcal{O}_{\theta_{\nu_1,\nu_2}}(X, x)$ *such that* $f(U) \subseteq V$.
(4) For each $x \in X$ *and each* $V \in \mathcal{O}_{r_{\upsilon_1,\upsilon_2}}(Y, f(x))$*, there exists a* ν_1*-open set* U *containing* x *such that* $f(c_{\nu_2}(U)) \subseteq V$.
(5) $f(\gamma_{\theta_{\nu_1,\nu_2}}(A)) \subseteq c_{\delta_{\upsilon_1,\upsilon_2}}(f(A))$ *for every* $A \subseteq X$.
(6) $\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}\left(c_{\delta_{\upsilon_1,\upsilon_2}}(B)\right)$ *for every* $B \subseteq Y$.

Proof. (1) $\Rightarrow$ (2) : Let $V \in \delta_{\upsilon_1,\upsilon_2}$and $x \in f^{\leftarrow}(V)$. By (1) $f^{\leftarrow}(V)$ is θ_{ν_1,ν_2}-open. Take $U = f^{\leftarrow}(V)$; then $U \in \mathcal{O}_{\theta_{\nu_1,\nu_2}}(X, x)$ and satisfies $f(U) \subseteq V$.
(2) $\Rightarrow$ (3) : Clear from Lemma 2.1(2).
(3) $\Rightarrow$ (4) : Let V be $r_{\upsilon_1,\upsilon_2}$-open in Y and $x \in f^{\leftarrow}(V)$. (3) implies that there exists a $W \in \mathcal{O}_{\theta_{\nu_1,\nu_2}}(X, x)$ such that $f(W) \subseteq V$. θ_{ν_1,ν_2}-openness of W implies that there exists a ν_1-open set U containing x such that $x \in U \subseteq c_{\nu_2}(U) \subseteq W$, so we have $f(c_{\nu_2}(U)) \subseteq f(W) \subseteq V$.
(4) $\Rightarrow$ (5) : Let $A \subseteq X$ and assume $y \in f(\gamma_{\theta_{\upsilon_1,\upsilon_2}}(A))$, then there exists $x \in \gamma_{\theta_{\upsilon_1,\upsilon_2}}(A)$ such that $y = f(x)$. Since $x \in \gamma_{\theta_{\upsilon_1,\upsilon_2}}(A)$, we have $A \cap c_{\nu_2}(U) \neq \emptyset$ for every ν_1-*open set* U *containing* x. By (4) *for every* $V \in \mathcal{O}_{r_{\upsilon_1,\upsilon_2}}(Y, f(x))$ *there exists a* ν_1*-open set* U *containing* x *such that* $f(c_{\nu_2}(U)) \subseteq V$, then we have $V \cap f(A) \neq \emptyset$. Hence $f(x) \in c_{\delta_{\upsilon_1,\upsilon_2}}(f(A))$.
(5) $\Rightarrow$ (6) : Taking $A = f^{\leftarrow}(B)$ in (5) we have $f(\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(B))) \subseteq c_{\delta_{\upsilon_1,\upsilon_2}}(f(f^{\leftarrow}(B)))$ $\subseteq c_{\delta_{\upsilon_1,\upsilon_2}}(B)$ and this gives $\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}\left(f(\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(B)))\right) \subseteq f^{\leftarrow}\left(c_{\delta_{\upsilon_1,\upsilon_2}}(B)\right)$ hence we get $\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}\left(c_{\delta_{\upsilon_1,\upsilon_2}}(B)\right)$.
(6) $\Rightarrow$ (1) : Let $M \in \delta_{\upsilon_1,\upsilon_2}$, then $Y \setminus M$ is $\delta_{\upsilon_1,\upsilon_2}$-closed set in Y, and since for arbitrary $C \subseteq Y$, $c_{\delta_{\upsilon_1,\upsilon_2}}(C)$ is $\delta_{\upsilon_1,\upsilon_2}$-closed set in Y, we have $c_{\delta_{\upsilon_1,\upsilon_2}}(Y \setminus M) = Y \setminus M$. Taking $B = Y \setminus M$ in (6) we get $\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(Y \setminus M)) \subseteq f^{\leftarrow}\left(c_{\delta_{\upsilon_1,\upsilon_2}}(Y \setminus M)\right) = f^{\leftarrow}(Y \setminus M)$ then $\gamma_{\theta_{\nu_1,\nu_2}}(f^{\leftarrow}(Y \setminus M)) = f^{\leftarrow}(Y \setminus M)$ so $f^{\leftarrow}(Y \setminus M)$ is θ_{ν_1,ν_2}-closed. Then we have $f^{\leftarrow}(M) \in \theta_{\nu_1,\nu_2}$. □

Definition 3.7. [10] Let $(X; \nu_1, \nu_2)$ be a BIGTS's on a set X ($\neq \emptyset$) and $A \subseteq X$. Then X is said to be (ν_1, ν_2)-almost regular if for $x \in X$ and an r_{ν_1,ν_2}-closed set F with $x \notin F$, there exist $U \in \nu_1$, $V \in \nu_2$ such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.

Theorem 3.10. [10] *Let* $(X; \nu_1, \nu_2)$ *be a BIGTS's on a set* X *(*$\neq \emptyset$*). If* X *is* (ν_1, ν_2)*-almost regular, every* r_{ν_1,ν_2}*-open set is* θ_{ν_1,ν_2}*-open.*

Theorem 3.11. *Let* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$ *be BIGTS's; let* $f : X \to Y$ *. If* X *is* (ν_1, ν_2)*-almost regular and* Y *is* (υ_1, υ_2)*-almost regular, then the following statements are equivalent:*
(1) f *is* $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$*-c.*
(2) f *is* $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(3) f *is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(4) f *is* $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$*-c.*

Proof. This is clear by Theorem 3. 10 of [10]. □

Theorem 3.12. *For BIGTS's* $(X; \nu_1, \nu_2)$*,* $(Y; \upsilon_1, \upsilon_2)$ *and* $(Z; \sigma_1, \sigma_2)$ *if* $f : X \to Y$ *is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.,* $g : Y \to Z$ *is* $(\delta_{\upsilon_1,\upsilon_2} : \theta_{\sigma_1,\sigma_2})$*-c. and* $(Y; \upsilon_1, \upsilon_2)$ *is* (υ_1, υ_2)*-almost regular then* $g \circ f : X \to Z$ *is* $(\delta_{\nu_1,\nu_2} : \theta_{\sigma_1,\sigma_2})$*-c.*

Proof. This is clear. □

4. COMPARISONS

Theorem 4.13. *For BIGTS's* $(X; \nu_1, \nu_2)$ *and* $(Y; \upsilon_1, \upsilon_2)$ *if the map* $f : (X; \nu_1, \nu_2) \to (Y; \upsilon_1, \upsilon_2)$ *is;*

(1) (ν_1, υ_1)*-c. and* (ν_2, υ_2) *-c., then it is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(2) $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$*-c., then it is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(3) $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c. then it is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(4) $(\delta_{\nu_1,\upsilon_2}, \theta_{\upsilon_1,\upsilon_2})$*-c. then it is faintly* $(\nu_1 : \upsilon_1\upsilon_2)$*-c.*
(5) $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$*-c., then it is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*
(6) weakly $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$*-c., then it is* $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$*-c.*

Proof. (1) Suppose that $V \in \theta_{\upsilon_1,\upsilon_2}$ and $f : X \to Y$ is (ν_1, υ_1)-c. and (ν_2, υ_2)-c. Then for $x \in f^{\leftarrow}(V)(\subseteq X)$ we have $y = f(x) \in V$ and there is a $W \in \upsilon_1$ satisfying $y \in W \subseteq c_{\upsilon_2}(W) \subseteq V$. Therefore it is true that $x \in f^{\leftarrow}(W) \subseteq f^{\leftarrow}(c_{\upsilon_2}(W))) \subseteq f^{\leftarrow}(V)$ and using $f^{\leftarrow}(W) \in \nu_1$ and $f^{\leftarrow}(c_{\upsilon_2}(W)))$ is ν_2-closed, we have $x \in f^{\leftarrow}(W) = i_{\nu_1}(f^{\leftarrow}(W))$ $\subseteq i_{\nu_1}(f^{\leftarrow}(c_{\upsilon_2}(W)))) \subseteq f^{\leftarrow}(c_{\upsilon_2}(W))) \subseteq f^{\leftarrow}(V)$ and this gives $f^{\leftarrow}(V)$ is δ_{ν_1,ν_2}-open.
(2) Let f be $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-c. and $W \in \theta_{\upsilon_1,\upsilon_2}$, then the containment $\theta_{\upsilon_1,\upsilon_2} \subseteq \delta_{\upsilon_1,\upsilon_2}$ gives $W \in \delta_{\upsilon_1,\upsilon_2}$. $(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity of f is gives $f^{\leftarrow}(W) \in \delta_{\nu_1,\nu_2}$.
(3) Let f be $(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c. and $W \in \theta_{\upsilon_1,\upsilon_2}$, then $f^{\leftarrow}(W) \in \theta_{\nu_1,\nu_2}$ and the containment $\theta_{\nu_1,\upsilon_2} \subseteq \delta_{\nu_1,\nu_2}$ implies $f^{\leftarrow}(W) \in \delta_{\nu_1,\nu_2}$.
(4) Let $f : X \to Y$ be $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c., then for every $W \in \theta_{\upsilon_1,\upsilon_2}$ it is true that $f^{\leftarrow}(W) \in \delta_{\nu_1,\nu_2}$ and from the containment $\delta_{\nu_1,\nu_2} \subseteq \nu_1$ it is clear that f is faintly $(\nu_1 : \upsilon_1\upsilon_2)$ -c.
(5) Let f be $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-c. and $W \in \theta_{\upsilon_1,\upsilon_2}$, then the containment $\theta_{\upsilon_1,\upsilon_2} \subseteq \delta_{\upsilon_1,\upsilon_2}$ gives $W \in \delta_{\upsilon_1,\upsilon_2}$. From the $(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$-continuity of f we have $f^{\leftarrow}(W) \in \theta_{\nu_1,\nu_2}$, again the containment $\theta_{\nu_1,\nu_2} \subseteq \delta_{\nu_1,\nu_2}$ gives $f^{\leftarrow}(W) \in \delta_{\nu_1,\nu_2}$.
(6) Let $f : X \to Y$ be weakly $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$-c., then for every $W \in \theta_{\upsilon_1,\upsilon_2}$ by Corollary 3.1, it is true that $f^{\leftarrow}(W) \in \delta_{\nu_1,\nu_2}$, hence f is $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c. □

Remark 4.2. For a map $f : (X; \nu_1, \nu_2) \to (Y; \upsilon_1, \upsilon_2)$ on BIGTS's, the following implications are valid:

(ν_1, υ_1)-c. and (ν_2, υ_2)-c.		$(\theta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$ -c.		faintly $(\nu_1 : \upsilon_1\upsilon_2)$-c.
$\downarrow\not\uparrow$	$\nearrow\not\swarrow$	$\downarrow\not\uparrow$	$\nearrow\not\swarrow$	
$(\theta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$ -c.	$\rightarrow$ $\not\leftarrow$	$(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$ -c.	$\not\rightarrow$ $\leftarrow$	weakly $(\delta_{\nu_1,\nu_2} : \upsilon_1\upsilon_2)$ -c.
	$\searrow\not\nwarrow$	$\uparrow\not\downarrow$		
		$(\delta_{\nu_1,\nu_2} : \delta_{\upsilon_1,\upsilon_2})$ -c.		

In the following examples, we show that in general, converse implications do not have to be true.

Example 4.1. Consider two generalized topologies $\nu_1 = \{\emptyset, \mathbb{R}\setminus\{3\}, \mathbb{R}\setminus\{2,3\}\}$ and $\nu_2 = \{\emptyset, \{2,3\}\}$ on $\mathbb{R}$. Then $\delta_{\nu_1,\nu_2} = \{\emptyset, \mathbb{R}\setminus\{3\}, \mathbb{R}\setminus\{2,3\}\}$, $\theta_{\nu_1,\nu_2} = \{\emptyset, \mathbb{R}\setminus\{2,3\}\}$. Again, let $\upsilon_1 = \{\emptyset, \mathbb{R}\setminus\{2,3\}\}$ and $\upsilon_2 = \{\emptyset, \{2,3\}\}$ be two GT's on $\mathbb{R}$. Then we have $\delta_{\upsilon_1,\upsilon_2} = \theta_{\upsilon_1,\upsilon_2} = \{\emptyset, \mathbb{R}\setminus\{2,3\}\}$.

- **(1):** The identity map f:$(\mathbb{R}; \nu_1, \nu_2) \to (\mathbb{R}; \nu_1, \nu_2)$ is, $(\delta_{\nu_1,\nu_2}, \theta_{\nu_1,\nu_2})$-c., $(\theta_{\nu_1,\nu_2}, \theta_{\nu_1,\nu_2})$-c. and $(\delta_{\nu_1,\nu_2}, \delta_{\nu_1,\nu_2})$-c. but it is not $(\theta_{\nu_1,\nu_2}, \delta_{\nu_1,\nu_2})$-c.
- **(2):** The identity map h:$(\mathbb{R}; \upsilon_1, \upsilon_2) \to (\mathbb{R}; \nu_1, \nu_2)$ is $(\delta_{\upsilon_1,\upsilon_2} : \theta_{\nu_1,\nu_2})$-c. and but f is not $(\delta_{\upsilon_1,\upsilon_2} : \delta_{\nu_1,\nu_2})$-c., f is not $(\theta_{\upsilon_1,\upsilon_2} : \delta_{\nu_1,\nu_2})$-c.

Example 4.2. Consider a generalized topology $\nu_1 (= \nu_2) = \{\emptyset, \mathbb{R}\setminus\{0,1\}, \mathbb{R}\setminus\{-1,0,1\}\}$ on $\mathbb{R}$ and two generalized topologies $\upsilon_1 = \{\emptyset, \mathbb{R}\setminus\{0,1\}, \mathbb{R}\setminus\{-1,0,1\}\}$ and $\upsilon_2 = \{\emptyset, \{0,1\}, \mathbb{R}\setminus\{0,1\}, \{-1,0,1\}, \mathbb{R}\}$ on $\mathbb{R}$. Then $\delta_{\nu_1,\nu_2} (= \delta(\nu_1)) = \{\emptyset, \mathbb{R}\setminus\{0,1\}\}$ and $\theta_{\upsilon_1,\upsilon_2} = \{\emptyset, \mathbb{R}\setminus\{0,1\}, \mathbb{R}\setminus\{-1,0,1\}\}$.

- **(1):** The identity map f:$(\mathbb{R}; \nu_1, \nu_2) \to (\mathbb{R}; \upsilon_1, \upsilon_2)$ is faintly $(\nu_1, \upsilon_1\upsilon_2)$-c., but since $f^{\leftarrow}(\mathbb{R}\setminus\{-1,0,1\}) = \mathbb{R}\setminus\{-1,0,1\} \notin \delta_{\nu_1,\nu_2}$, f is not $(\delta_{\nu_1,\nu_2} : \theta_{\upsilon_1,\upsilon_2})$-c. map.

(2): The identity map g:$(\mathbb{R};\upsilon_1,\upsilon_2)\to(\mathbb{R};\nu_1,\nu_2)$ is $(\theta_{\upsilon_1,\upsilon_2}:\delta_{\nu_1,\nu_2})$ -c. but since $g^{\leftarrow}(\mathbb{R}\setminus\{-1,0,1\})=\mathbb{R}\setminus\{-1,0,1\}\notin\upsilon_2$ it is not (υ_2,ν_2)-c.

(3): The identity map h:$(\mathbb{R};\upsilon_1,\upsilon_2)\to(\mathbb{R};\upsilon_1,\upsilon_2)$ is *weakly* $(\delta_{\upsilon_1,\upsilon_2}:\upsilon_1\upsilon_2)$-c.

Example 4.3. Let $X=\{1,2,3,4\}$ let $\nu_1=\{\emptyset,\{1,2\},\{3,4\},X\}$ $\nu_2=\{\emptyset,\{1,2\}\}$ be two GT's on X. Then we have $\delta_{\nu_1,\nu_2}=\theta_{\nu_1,\nu_2}=\{\emptyset,\{3,4\},X\}$. Let $Y=\{1,2,3\}$ and let us consider two generalized topologies $\upsilon_1=\{\emptyset,\{1\},\{1,3\},\{2,3\},Y\}$ and $\upsilon_2=\{\emptyset,\{1\},\{3\},\{1,3\},Y\}$ on Y. Then $\delta_{\upsilon_1,\upsilon_2}=\{\emptyset,\{1\},\{2,3\},Y\}$, $\theta_{\upsilon_1,\upsilon_2}=\{\emptyset,\{2,3\},Y\}$. Consider a map f: $(X;\nu_1,\nu_2)\to(Y;\upsilon_1,\upsilon_2)$ defined as $f(1)=2$, $f(2)=1$, $f(3)=2$, $f(4)=3$. Then f is $(\delta_{\nu_1,\nu_2},\theta_{\upsilon_1,\upsilon_2})$-c., but not *weakly* $(\delta_{\nu_1,\nu_2}:\upsilon_1\upsilon_2)$-c. (there is no $U\in\nu_1$ satisfying $f(i_{\nu_1}(c_{\nu_2}(U)))\subseteq c_{\upsilon_2}(\{1\})$).

References

[1] Bayhan, S., Kanibir, A. and Reilly, I. L., *On functions between generalized topological spaces*, Appl. Gen. Topol., **14** (2013), No. 2, 195–203

[2] Cammaroto, F. and Noiri, T., *On weakly θ-continuous functions*, Mat.Vesnik, **38** (1986), No. 1, 33–44

[3] Császár, Á., *Generalized topology, generalized continuity*, Acta Math. Hungar., **96** (2002), 351–357

[4] Császár, Á., *Separation axioms for generalized topologies*, Acta Math. Hungar., **104** (2004), 63–69

[5] Császár, Á., *Generalized open sets in generalized topologies*, Acta Math. Hungar., **106** (2005), 53–66

[6] Császár, Á., *δ- and θ-modifications of generalized topologies*, Acta Math. Hungar., **120** (2008), 275–279

[7] Császár, Á. and Makai, E., *Further remarks on δ- and θ-modifications*, Acta Math. Hungar., **123** (2009), 223–228

[8] Khedr, F. H. and Al-Shibani, A. M., *Weakly θ-continuuous mappings in bitopological spaces*, Bull. Fac. Sci., Assiut Univ. C, **23** (1994), No. 1, 105–118

[9] Min, W. K., *Mixed weak continuity on generalized topological spaces*, Acta Math. Hungar., **132** (2011), No. 4, 339–347

[10] Min, W. K., *A note on δ- and θ-modifications*, Acta Math. Hungar., **132** (2011), No. 1-2, 107–112

[11] Min, W. K., *Mixed θ-continuity on generalized topological spaces*, Math. Comput. Modelling, **54** (2011), No. 11-12, 2597–2601

[12] Santoro, G., *On θδ-continuous functions*, Zb. Rad. No. 6, part 2 (1992), 259–268

[13] Veličko, N. V., *H-closed topological spaces*. Mat. Sb., **70** (1966), 98–112

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