CREAT. MATH. INFORM. **28** (2019), No. 1, 49 - 52
Online version at http://creative-mathematics.ubm.ro/
Print Edition: ISSN 1584 - 286X Online Edition: ISSN 1843 - 441X

Geometrical characteristics of the stability domain in the restricted problem of eight bodies

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ABSTRACT. The eight-body Newtonian problem is studied. Applying the symbolic calculation system Mathematica the stationary solutions, their stability in numerical form and the geometric characteristics of the stability domain are studied.

1. INTRODUCTION

The Newtonian problem of n bodies was formulated by I. Newton more than 300 years ago and consists in studying the movement of bodies in the Newtonian gravitational field. So far, it has not been fully resolved, although so many attempts have been made to determine its exact solutions.

The restricted problem of n-bodies is a typical Hamiltonian problem. The problem of the stability of his stationary solutions in the Lyapunov sense can be solved only within the KAM theory (theory of the conditional-periodic solutions of A. N. Kolmogorov, V. I. Arnold, J. Moser). This involves carrying out a series of voluminous analytical transformations of Hamiltonian (see[1], [3]). The development of new computer technologies has provided the opportunity to otherwise approach the problem of n-bodies.

In the present article, applying the symbolic calculation system Mathematica, we determine the existence of the stationary points, their stability in the Lyapunov sence and examine the geometrical characteristics of the domain of the stability of the restricted and planar Newtonian problem of eight bodies with incomplete symmetry. As with the analytical methods we can't solve these problems, a number of numerical experiments have been performed. They provide new information about the trajectory comportment in the neighborhood of the equilibrium points. These transformations and calculations were obtained by applying the possibilities of the symbolic calculation system Mathematica (see [1], [2], [3], [4]).

2. DESCRIPTION OF THE CONFIGURATION

Assume that in a the non-inertial space there is the motion of seven bodies P_0, P_1, P_2, P_3, P_4, P_5, P_6, with the masses m_0, m_1, m_2, m_3, m_4, m_5, m_6, which attract each other in accordance with the law of universal attraction. We will investigate the planar dynamic pattern formed by a square in the vertices of which the points P_1, P_2, P_3, P_4, are located, the other two points P_5, P_6, having the masses $m_5 = m_6$ are on the diagonal P_1P_3 of the square at equal distances from point P_0, in around which this configuration rotates with a constant angular velocity ω which is determined from the model parameters.

We can assume that $P_1(1,1)$, $P_2(-1,1)$, $P_3(-1,-1)$, $P_4(1,-1)$, $P_5(\alpha,\alpha)$, $P_6(-\alpha,-\alpha)$, $f = 1$, $m_0 = 1$, $m_5 = m_6$. Then out of the differential equations of the motion obtain the

Received: 17.07.2018. In revised form: 03.12.2018. Accepted: 10.12.2018
2010 *Mathematics Subject Classification.* 70F10, 34D35.
Key words and phrases. *configuration, differential equations of motion, stationary solutions, linear stability.*

existence conditions of this configuration

$$m_1 = m_3,\, m_2 = m_4 = f_1\left(m_1, \alpha\right),\, m_5 = m_6 = f_2\left(m_1, \alpha\right),\, \omega^2 = f_3\left(m_1, \alpha\right). \tag{2.1}$$

It will be studied the motion of the body P with a infinitely small mass (the so-called passive gravitational body) in the gravitational field by the given seven bodies. It is known that this dynamic model generates a new problem - the restricted problem of eight bodies.

3. Differential equations of the restricted problem of eight bodies

In a non-inertial space P_0xyz there have the motion of eight bodies P_0, P_1, P_2, P_3, P_4, P_5, P_6, P. Differential equations of the body $P(x, y, z)$ which gravitates passively in the field of the other seven bodies in the rotating space have the form (see [1], [3]):

$$\begin{cases} \dfrac{d^2X}{dt^2} - 2\omega\dfrac{dY}{dt} = \omega^2 X - \dfrac{fm_0X}{r^3} + \dfrac{\partial R}{\partial X}, \\ \dfrac{d^2Y}{dt^2} + 2\omega\dfrac{dX}{dt} = \omega^2 Y - \dfrac{fm_0Y}{r^3} + \dfrac{\partial R}{\partial Y}, \\ \dfrac{d^2Z}{dt^2} = -\dfrac{fm_0Z}{r^3} + \dfrac{\partial R}{\partial Z}, \end{cases} \tag{3.2}$$

where

$$\begin{cases} R = f\sum\limits_{j=1}^{6} m_j\left(\dfrac{1}{\Delta_{kj}} - \dfrac{XX_j + YY_j + ZZ_j}{r_j^3}\right), \\ \Delta_j^2 = \left(X_j - X\right)^2 + \left(Y_j - Y\right)^2 + \left(Z_j - Z\right)^2, \\ r_j^2 = X_j^2 + Y_j^2 + Z_j^2,\ r^2 = X^2 + Y^2 + Z^2, \\ j = 1, 2, ..., 6. \end{cases} \tag{3.3}$$

$(X_j, Y_j, Z_j = 0)$, are the respective coordinates of the bodies P_1, P_2, P_3, P_4, P_5, P_6, and $m_2 = m_4 = f_1(m_1, \alpha)$, $m_5 = m_6 = f_2(m_1, \alpha)$, $\omega^2 = f_3(m_1, \alpha)$ are determined by the existence conditions of the studied configuration.

Using the graphical possibilities of the system Mathematica, the stationary points of the system (3.2) were determined. For this we use a algorithm similar to the algorithms from [1], [3] for other configurations. For concrete values of m_1 and α we obtain concrete stationary points. Moreover, we obtain the values of the geometrical parameter α for which the stationary points is linearly stable.

4. Study of the stability in the numerical form

We will study the stationary point

$$S\left(1.41168, -0.12379\right), \tag{4.4}$$

determined for m_1 = 0.01 and α = 0.8584, $Z = 0$. He is linearly stable, because the eigenvalues of the matrix of linearized system (3.2) have the real part null: $\pm 0.49471i$; $\pm 0.32201i$.

Using the system Mathematica, we can solve the differential equations (3.2) with the initial data (4.4) for over a fairly long of time in the form of interpolation functions:

$$S1 = NDSolve[\{x''\left[t\right] - 2y'\left[t\right] == f, x\left[0\right] == x_1, x'\left[0\right] == 0,$$
$$y''\left[t\right] + 2x'\left[t\right] == g, y\left[0\right] == y_1, y'\left[0\right] == 0\},$$
$$\left\{x, y\right\}, \left\{t, 0, 10000\right\}].$$

By f, g were noted the respective right-hand side of equations (3.2), x_1, y_1 are the coordinates of point S.

The graphs of the obtained functions are constructed using the instruction:

$$ParametricPlot[Evaluate[\{x[t], y[t]\}/.S1], \{t, 0, t_1\},$$

$$AxesLabel \rightarrow \{''x[t]'','' y[t]''\}, AxesOrigin \rightarrow \{x_1, y_1\}].$$

The solutions of differential equations are not obtained in the tables form, but as interpolation functions. Their graphs can be construct for different integration intervals. In the fig.$1-2$ are displayed their graphs for time intervals: $0 < t < 250, 0 < t < 1000$. In these figures, the origin is translated in the point S.

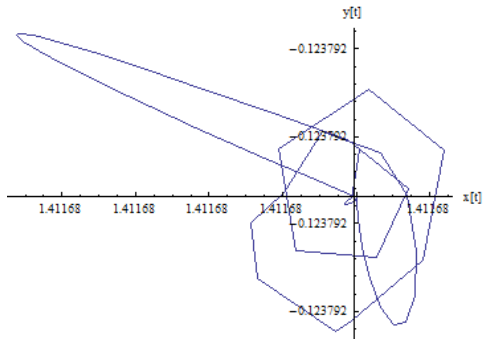

FIGURE 1. Geometrical representation of $x[t], y[t]$ for $0 < t < 250$

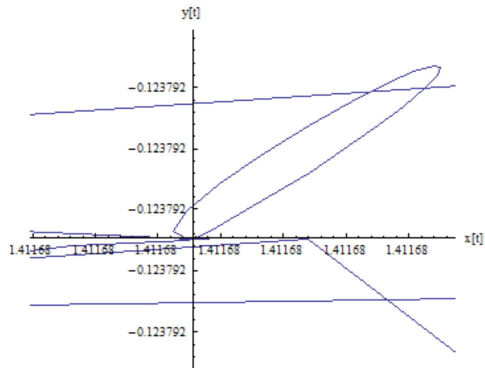

FIGURE 2. Geometrical representation of $x[t], y[t]$ for $0 < t < 1000$

Analyzing the drawings we note that the trajectory does not depart far from the equilibrium point S.

Let $\Delta r\,(t)$ be the local distance from the equilibrium point to the point on the trajectory. We will examine comportment of $\Delta r\,(t)$ for the same time intervals as from fig. $1-2$.

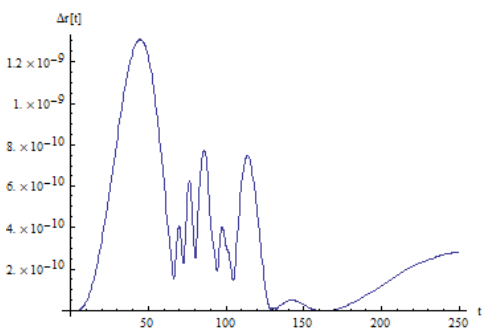

FIGURE 3. Geometric representation of $\Delta r\,(t)$ for $0 < t < 250$

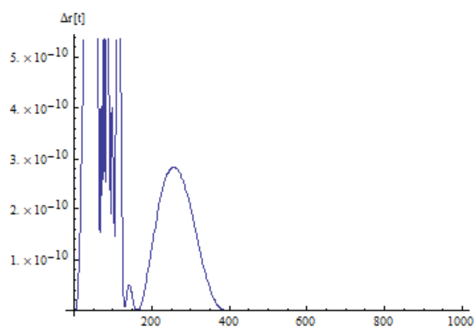

FIGURE 4. Geometric representation of $\Delta r\,(t)$ for $0 < t < 1000$

Out of fig. 3-4 it is seen that the distance of the trajectory from the equilibrium point is quite small. At long intervals of time the trajectory tends to keep the same distance. So, based on these numerical experiments, we can admit that the point S is asymptotic stable. This obtained result is not in contradiction with the theoretical researches.

5. Conclusions

The numerical solving procedures of the differential equations of motion in the restricted eight bodies problem allow to qualitatively estimate the size and form of the stability domain for sufficiently large time intervals. The results of the numerical study aren't in contradiction with the theoretical researches.

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