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Comments on "A two-stage supply chain problem with fixed costs: An ant colony optimization approach" by Hong et al. International Journal of Production Economics (2018)

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ABSTRACT. The two-stage supply chain problem with fixed costs consists of designing a mimimum distribution cost configuration of the manufacturers, distribution centers and retailers in a distribution network, satisfying the capacity constraints of the manufacturers and distribution centers so as to meet the retailers specific demands. The aim of this work is to pinpoint some inaccuracies regarding the paper entitled "A two-stage supply chain problem with fixed costs: An ant colony optimization approach" by Hong et al. published in International Journal of Production Economics, Vol. **204**, pp. 214–226 (2018) and to propose a valid mixed integer programming based mathematical model of the problem. The comments are related to the mathematical formulation proposed by Hong et al. and the considered test instances.

1. INTRODUCTION

Supply chains (SCs) are defined as worldwide networks wherein the following actors appear: supplier, manufacturers, distribution centers, retailers and customers and their main objective being the satisfaction of the customer requirements. In order to achieve an efficient and effective management of SC systems, the researchers emphasized on the transportation system design, as it plays an important and central role. A typical representation of a SC is as a form of multi-staged structure, while its optimal design has been recognized to be a NP-hard problem [1].

Different variants of the two-stage supply chain problem have been considered in the literature, depending on the characteristics of the transportation system which models real applications of the supply chain network design. We refer to Chen et al. [1], Pop et al. [3] and Hong et al. [2] for more information.

Hong et al. [2] proposed a particular supply chain network design problem, namely the two-stage transportation problem with fixed charge for opening the distribution centers and fixed transportation costs associated to the routes between manufacturers and distribution centers (DC's) and between DC's and retailers. In addition, they supposed that the total demands of the retailers might be fulfilled by each of the DCs. The considered variant can be defined as follows: given a set of m manufacturers, a set of d distribution centers (DC's) and a set of r retailers with the following properties:

- Each manufacturer $i \in \{1, ..., m\}$ has S_i units of supply, each DC_j, where $j \in \{1, ..., d\}$ has a given storage capacity SC_j and each retailer k has a demand D_k, where $k \in \{1, ..., r\}$.
- Each manufacturer may ship to any of the d distribution centers at a transportation cost c'_{ij} per unit from manufacturer i, where $i \in \{1, ..., m\}$, to DC_j, where $j \in \{1, ..., d\}$;

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- Each DC may ship to any of the r retailers at a transportation cost c''_{jk} per unit from DC_j, where $j \in \{1, ..., d\}$, to retailer k, where $k \in \{1, ..., r\}$;
- There exist fixed costs for opening the distribution centers denoted by f_j, where $j \in \{1, ..., d\}$ and fixed transportation costs from each manufacturer to each DC, denoted by f'_{ij}, where $i \in \{1, ..., m\}$ and $j \in \{1, ..., d\}$, and from each DC to each retailer, denoted by f''_{jk}, where $j \in \{1, ..., d\}$ and $k \in \{1, ..., r\}$.

The aim of the two-stage supply chain problem with fixed costs associated to the routes and for opening the distribution centers is to determine the distribution centers and the routes to be opened and corresponding shipment quantities on these routes, such that the customer demands are fulfilled, all shipment constraints are satisfied, and the total distribution costs are minimized.

The scope of this work is to show some inaccuracies appeared in the paper published by Hong et al. [2] and as well as to provide a valid mixed integer programming based mathematical model of the two-stage supply chain problem with fixed costs. The proposed model is tested on a set of 150 instances divided into three classes of problems: smaller, medium and large.

The paper is organized according to the following structure: the next section presents some inaccuracies appeared in the paper published by Hong et al. [2]. A valid mixed integer programming formulation of the two-stage supply chain problem with fixed costs is described in Section 3. Computational experiments and the achieved results are presented in Section 4, while in Section 5 some concluding results, as well as further remarks are presented.

2. Comments

In Hong et al. [2] the constraints (2) and (5) of the proposed mixed linear integer programming, page 218, are described as follows:

$$\sum_{i=1}^{m} x'_{ij} \leq S_i, \ \forall j \in \{1, ..., d\} \tag{2.1}$$

$$\sum_{k=1}^{y} D_k = SC_j, \ \forall j \in \{1, ..., d\} \tag{2.2}$$

The constraints (2.1) are not correct and should be replaced by the following constraints:

$$\sum_{j=1}^{d} x'_{ij} \leq S_i, \ \forall i \in \{1, ..., m\} \tag{2.3}$$

which are modeling the fact that the sum of delivered quantities from a given manufacturing plant to the distribution centers should be less or equal to the capacity of the manufacturing plant.

Regarding (2.2), these are not even constraints of the mathematical model because they do not contain any variables, there are just assumptions meaning that the total demands of the retailers might be fulfilled by each of the DCs.

Concerning the computational study reported by Hong et al. [2] we have the following observations:

1. The set of instances used in the computational experiments were generated randomly and belong to three classes of problems: *smaller* which consists of 2 manufacturing plants, 5 distribution centers and 10 retailers, *medium* which consists of

4 manufacturing plants, 8 distribution centers and 15 retailers and *larger* which consists of 6 manufacturing plants, 10 distribution centers and 20 retailers. All the considered test problem instances are fairly easy to solve to proven optimality.Because we did not have access to the data used in the computational experiments of Hong et al. [2], we generated as they have suggested and solved the valid mixed integer programming formulation of the problem with CPLEX (version 12.7.0) on a PC with i7 processor, 2.6GHZ. The small instances were solved within about 0.05 seconds and the larger instances were solved within 90 seconds, for further details we refer to Section 4.

2. The average data reported by Hong et al. [2] on page 223, Section 5.3 are different from those calculated from the vales reported in Tables 11b and 11c.
3. On page 220, the Hong et al. [2] mentioned that "the capacity of a distribution centre is assumed to be unlimited and the production capacity of a plant is designed to meet the total demand". These assumptions are not correct. If the DC's are unlimited the mathematical model defined by the authors is infeasible. It is correct to suppose that the DC's have a large capacity equal to the total demand of the retailers and the total production capacity of the manufacturing plants is designed to meet the total demand of the retailers.

3. A VALID MATHEMATICAL MODEL OF THE TWO-STAGE SUPPLY CHAIN PROBLEM WITH FIXED COSTS

With the purpose of formulating the two-stage supply chain problem with fixed costs as a mixed integer program, we introduce the following decision variables:

- Linear variables:
  - x'_{ij} specifying the number of units shipped from plant i to the distribution center j;
  - x''_{jk} specifying the number of units shipped from distribution center j to the retailer k;
- Binary variables:
  - y'_{ij} specifying if there are units shipped from plant i to the distribution center j ($y'_{ij} = 1$, if $x'_{ij} > 0$ and $y'_{ij} = 0$, otherwise);
  - y''_{jk} specifying if there are units shipped from distribution center j to the retailer k ($y''_{jk} = 1$, if $x''_{jk} > 0$ and $y''_{jk} = 0$, otherwise);
  - z_j specifying if the distribution center j is open ($z_j = 1$, if the distribution center j is open and $z_j = 0$, otherwise).

We can express the two-stage supply chain problem with fixed costs as the following mixed integer programming problem:

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{d}(c'_{ij}x'_{ij} + f'_{ij}y'_{ij}) + \sum_{j=1}^{d}\sum_{k=1}^{r}(c''_{jk}x''_{jk} + f''_{jk}y''_{jk}) + \sum_{j=1}^{d} f_j z_j$$

$$s.t. \quad \sum_{j=1}^{d} x'_{ij} \leq S_i, \qquad \forall i \in \{1, ..., m\} \tag{3.4}$$

$$\sum_{j=1}^{d} x''_{jk} = D_k, \qquad \forall k \in \{1, ..., r\} \tag{3.5}$$

$$\sum_{i=1}^{m} x'_{ij} = \sum_{k=1}^{r} x''_{jk}, \ \forall j \in \{1, ..., d\} \tag{3.6}$$

$$\sum_{k=1}^{r} x''_{jk} \leq SC_j \cdot z_j, \ \ \forall j \in \{1, ..., d\} \tag{3.7}$$

$$x'_{ij} \geq 0, \qquad \forall i \in \{1, ..., m\}, \ j \in \{1, ..., d\} \tag{3.8}$$

$$x''_{jk} \geq 0, \qquad \forall j \in \{1, ..., d\}, \ k \in \{1, ..., r\} \tag{3.9}$$

$$y'_{ij} \in \{0, 1\}, \qquad \forall i \in \{1, ..., m\}, \ j \in \{1, ..., d\} \tag{3.10}$$

$$y''_{jk} \in \{0, 1\}, \qquad \forall j \in \{1, ..., d\}, \ k \in \{1, ..., r\} \tag{3.11}$$

$$z_j \in \{0, 1\}, \qquad \forall j \in \{1, ..., d\} \tag{3.12}$$

Our objective is to minimize the total transportation cost including the unit transportation costs and the fixed costs (for opening DC's and associated to the routes). Constraints (3.4) guarantee that the capacity of the manufacturers is not exceeded. Constraints (3.5) guarantee that the customer demand is fulfilled. Constraints (3.6) are the flow conservation conditions and they guarantee that the units received by a distribution center from manufacturers are equal to the units shipped from the distribution center to the retailers. Constraints (3.7) guarantee that storage capacity of the distribution center's is not exceeded. Finally, the last constraints set the ranges of the decision variables.

Regarding the considered illustrative example which consists of 2 manufacturing plants, 4 distribution centers and 6 retailers, we refer to the work of Hong et al. [2] for more information concerning the way there were chosen the demands of the retailers, the production capacity of the plants, the opening costs of the distribution centers and the fixed and unit transportation costs from plants to distribution centers, respectively from distribution centers to retailers.

We solved this example using the proposed mixed integer programming formulation of the problem with CPLEX and in Figure 1 we present an illustration and the obtained optimal solution.

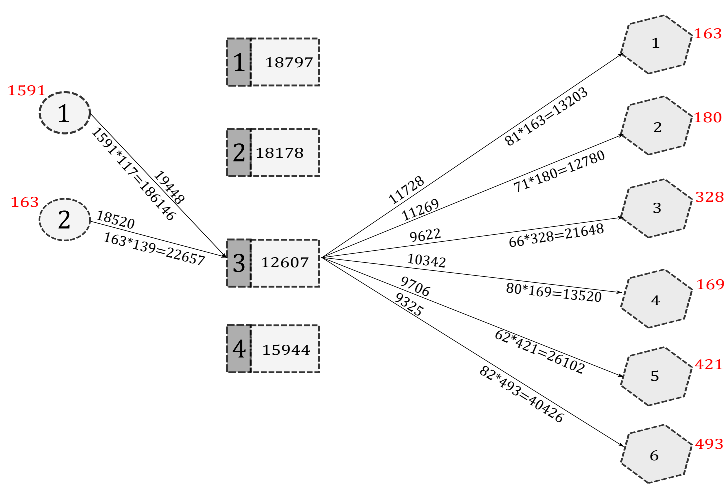

FIGURE 1. An illustration and the obtained optimal solution

Solving this example to optimality using our model by means of CPLEX just took 0.05 seconds and 30 iterations, in contrast to model proposed by Hong et al. [2] which used LINGO solver in order to obtain the optimal solution of cost 449050 within 93 iterations and the proposed ant colony approach which provided an suboptimal solution of cost 476138 within 0.24 seconds.

4. Computational experiments

The proposed mixed integer programming model of the two-stage supply chain problem with fixed costs was implemented in CPLEX 12.7 and has been tested on a PC with i7 processor, 2.6GHz.

Because we did not have access to the data used in the computational experiments of Hong et al. [2], we generated as they have suggested and we considered 150 test instances classified into three problem classes: smaller which consists of 2 manufacturing plants, 5 distribution centers and 10 retailers, medium which consists of 4 manufacturing plants, 8 distribution centers and 15 retailers and large which consists of 6 manufacturing plants, 10 distribution centers and 20 retailers. All the instances used in our computational experiments are available at the address: `http://dx.doi.org/10.17632/tnsw24zgmk.2`, [4].

Table 1 3 summarize the computational experiments performed for solving the considered instances using CPLEX 12.7. The first and the fifth column indicate the name of the instance, the second and the sixth column show the cost of the achieved solution, the third, fourth, seventh and eighth columns contain the necessary number of iterations and the computational times in order to achieve the corresponding solutions.

TABLE 1. Experimental results in the case of small instances

Instance	Cost	Iterations	Time(s)	Instance	Cost	Iterations	Time(s)
small_01	153202	73	0.078	small_26	147448	68	0.047
small_02	123809	58	0.047	small_27	129018	37	0.016
small_03	123206	72	0.047	small_28	110212	78	0.031
small_04	128373	33	0.016	small_29	118859	34	0.016
small_05	112287	73	0.047	small_30	137594	56	0.047
small_06	116087	107	0.063	small_31	143425	55	0.031
small_07	121482	107	0.047	small_32	125740	71	0.031
small_08	132550	47	0.031	small_33	111767	55	0.047
small_09	130213	59	0.031	small_34	117009	46	0.016
small_10	110925	52	0.031	small_35	107492	49	0.015
small_11	143985	58	0.031	small_36	110109	56	0.031
small_12	143086	62	0.031	small_37	141582	42	0.016
small_13	147201	94	0.047	small_38	113649	41	0.032
small_14	123451	94	0.046	small_39	118902	54	0.032
small_15	96593	44	0.016	small_40	120134	81	0.047
small_16	139162	81	0.047	small_41	115041	47	0.062
small_17	125680	47	0.031	small_42	150258	69	0.078
small_18	128300	60	0.032	small_43	131099	68	0.031
small_19	141352	66	0.047	small_44	170979	31	0.016
small_20	166768	46	0.016	small_45	124716	55	0.047
small_21	121662	56	0.047	small_46	138829	70	0.031
small_22	121141	41	0.015	small_47	140716	27	0.01
small_23	123189	69	0.063	small_48	87977	38	0.015
small_24	143520	49	0.062	small_49	153017	83	0.047
small_25	105190	69	0.031	small_50	141750	60	0.031

All the solutions reported in Tables 1 2 are the optimal one, while in Table 3 those with a computational time less then 90 seconds are optimal. For the other instances from Table 3, CPLEX was stopped after 90 seconds and in these cases the provided solutions produced a gap less than 1%.

TABLE 2. Experimental results in the case of medium instances

Instance	Cost	Iterations	Time(s)	Instance	Cost	Iterations	Time(s)
medium_01	477151	14222	0.344	medium_26	511471	1058	0.172
medium_02	573993	25078	0.5	medium_27	483214	39238	0.609
medium_03	490909	3130	0.235	medium_28	488269	4672	0.171
medium_04	515829	288	0.188	medium_29	477490	195979	2.875
medium_05	478893	2933	0.203	medium_30	424910	132587	2.391
medium_06	512521	7787	0.265	medium_31	458845	22067	0.359
medium_07	557735	77	0.062	medium_32	458035	3188	0.172
medium_08	484093	517	0.156	medium_33	569393	24234	0.344
medium_09	371166	3671	0.203	medium_34	454671	4363	0.188
medium_10	473509	5061	0.203	medium_35	568494	30545	0.422
medium_11	614210	9361	0.344	medium_36	496198	100374	1.813
medium_12	454787	28759	0.515	medium_37	535995	50505	0.672
medium_13	507734	1672	0.14	medium_38	505710	3898	0.187
medium_14	528003	123	0.078	medium_39	521957	17307	0.328
medium_15	536279	87	0.063	medium_40	506499	5093	0.188
medium_16	541936	3663	0.266	medium_41	366150	340	0.141
medium_17	438783	9538	0.25	medium_42	573072	95	0.063
medium_18	522471	174	0.094	medium_43	509418	221	0.109
medium_19	599152	8575	0.266	medium_44	448406	611	0.14
medium_20	474951	211	0.156	medium_45	431804	584	0.125
medium_21	380631	69645	1.359	medium_46	441069	268	0.125
medium_22	547807	92640	1.86	medium_47	414102	8529	0.235
medium_23	496648	262	0.11	medium_48	481981	57903	0.938
medium_24	561287	28817	0.484	medium_49	467138	3503	0.172
medium_25	531807	10331	0.203	medium_50	487939	3735	0.187

TABLE 3. Experimental results in the case of large instances

Instance	Cost	Iterations	Time(s)	Instance	Cost	Iterations	Time(s)
large_01	1388633	5018997	71.578	large_26	1364659	3809907	90.031
large_02	1542820	5080640	90.031	large_27	1328556	1481429	25.844
large_03	1312004	4450811	90.047	large_28	1181204	1742888	30.312
large_04	1382901	3988012	90.047	large_29	1483009	1007275	18.797
large_05	1211023	5294698	90.032	large_30	1406012	4110552	90.031
large_06	1151335	979211	15.547	large_31	1172779	2520206	43.64
large_07	1382812	3910544	90.031	large_32	1351554	4629204	90.031
large_08	1359964	4125854	90.031	large_33	1180790	3582980	46.829
large_09	1379187	4171515	90.047	large_34	1263040	4644046	90.031
large_10	1334627	1040107	17.968	large_35	1451735	4260054	90.032
large_11	1346373	4530789	90.031	large_36	1403634	4786641	90.031
large_12	1250390	1293359	21.125	large_37	1359081	3873681	90.031
large_13	1318204	783685	15.688	large_38	1453512	450293	9.343
large_14	1285857	5549697	84.875	large_39	1376291	3855391	90.031
large_15	1319339	602681	12.063	large_40	1321888	32405	0.531
large_16	1333217	472961	10.406	large_41	1325156	5084042	90.031
large_17	1349054	2021330	34.157	large_42	1344454	4371606	90.032
large_18	1499853	4736499	90.032	large_43	1341675	4128823	90.031
large_19	1286003	2580822	42.406	large_44	1274240	3976081	90.047
large_20	1444426	1264712	22.625	large_45	1247675	4622092	90.046
large_21	1368194	798565	13.953	large_46	1535077	4881453	90.031
large_22	1369308	4124503	69.437	large_47	1387158	2757750	39.218
large_23	1255037	770520	14.484	large_48	1333314	5975289	90.031
large_24	1374280	5503664	90.046	large_49	1436633	1786357	31.015
large_25	1527595	4601345	90.047	large_50	1386789	86104	2.156

In Table 4 we compare our novel mathematical model with the formulation provided by Hong et al. (Hong et al., 2018) and the ACO-based heuristic approach. In Table 4, we provided the following characteristics: the average computational times in seconds and the average number of iterations necessary to solve optimally the two-stage supply chain problem with fixed costs using our proposed mathematical model with CPLEX and the average results reported by Hong et al. (Hong et al., 2018) using LINGO, respectively the ACO-based heuristic approach.

TABLE 4. Comparison between our model solved with CPLEX, the model provided by Hong et al. solved with LINGO and the ACO-based heuristic approach

Problem	CPLEX		LINGO		ACO	
Instance set	Av. Iterations	Av. Time(s)	Av. Iterations	Av. Time(s)	Av. Iterations	Av. Time(s)
Smaller	59.76	0.03	568.7	-	-	103.86
Medium	20750.38	0.43	292472.6	710.46	-	175.79
Large	3203041.4	60.69	115243538.3	8191.46	-	262.22

In the case of the larger instances, there were 26 out of 50 instances in which CPLEX was interrupted after 90 seconds. The sign - means that the corresponding data have not been provided by Hong et al. [2]. The average data reported for LINGO and ACO-based heuristic approach were calculated from Tables 11a, 11b and 11c and are different from the values referred in section 5.3 (page 223) by Hong et al. [2].

Analyzing the computational results, we can observe that our proposed model outperforms in terms of computational times and number of iterations the model described by Hong et al. (Hong et al., 2018) and even the ACO-based heuristic approach, which according to the previously mentioned authors provided sub-optimal solutions with a gap of about 10% in average from the optimal solutions. For solving even larger instances CPLEX may encounter difficulties and the computational times will grow exponentially by increasing the dimension of the problem, therefore in this situation suitable approaches might be heuristic and metaheuristic algorithms.

5. CONCLUSIONS

In this paper we presented some inaccuracies appeared in the paper published by Hong et al. [2] and as well as we provided a valid mixed integer programming based mathematical model of the two-stage supply chain problem with fixed costs. Computational experiments were reported on a set of 150 instances. The obtained results prove the strength of our proposed model.

Finally, we present some future research directions. The proposed model can be used in cutting plane or decomposition approaches and can be combined with metaheuristics methods based on local search. Dealing with large size instances will require the design of heuristic or metaheuristic approaches, which within polynomial time may deliver good quality sub-optimal solutions.

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