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On equitable chromatic topological indices of some Mycielski graphs

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ABSTRACT. In recent years, the notion of chromatic Zagreb indices has been introduced and studied for certain basic graph classes, as a coloring parallel of different Zagreb indices. A proper coloring $\mathcal{C}$ of a graph G, which assigns colors to the vertices of G such that the numbers of vertices in any two color classes differ by at most one, is called an equitable coloring of G. In this paper, we introduce the equitable chromatic Zagreb indices and equitable chromatic irregularity indices of some special classes of graphs called Mycielski graphs of paths and cycles.

1. INTRODUCTION

In chemical graph theory, the topological indices of a graph may be considered as certain connectivity indices which are invariant under graph isomorphism. They are widely used in many areas. In particular, topological graph indices are long-established in mathematical chemistry as molecular descriptors calculated based on molecular structures of chemical objects.

The studies on topological indices of graphs commenced three decades ago, when the study on the dependence of total π- electron energy on molecular structures has been done in [5]. Zagreb indices of graphs are the first two topological indices calculated using vertex degrees. Over years, the research work formed around these concepts are massive. Recently a chromatic analogue of these indices, especially of Zagreb indices have been introduced in literature (see [8]).

For terms and definitions which are not introduced in this paper, we refer the reader to [6, 2, 3, 13]. Throughout our study, we consider $G = (V, E)$ as a finite, non-trivial, undirected, simple and connected graph.

A *graph coloring* is an assignment of colors or labels or weights to the vertices, edges and faces of a graph under consideration. Unless stated otherwise, in this paper, the graph coloring is meant to be an assignment of colors to the vertices of a graph subject to certain conditions. A *proper vertex coloring* of a graph G is an assignment $\varphi : V(G) \to \mathcal{C}$, where $\mathcal{C} = \{c_1, c_2, c_3, \ldots, c_\ell\}$ is a set of colors, such that adjacent vertices of G have different colors. This coloring is called an ℓ-coloring of the graph G. The *chromatic number* of a graph G, denoted by $\chi(G)$, is the minimum number of colors required in a proper coloring of the given graph.

The set of all vertices of G which have the color c_i is called the *color class* of that color c_i in G, denoted by $\mathcal{C}_i$. The cardinality of the color class of a color c_i is said to be the *strength* of that color in G and is denoted by $\theta(c_i)$. We can also define a function $\zeta_e : V(G) \to \{1, 2, 3, \ldots, \ell\}$ such that $\zeta_e(v_i) = s \iff \varphi_e(v_i) = c_s, c_s \in \mathcal{C}$. Also, we denote the number of edges with end points having colors c_t and c_s by η_{ts}, where $t < s, 1 \leq t, s \leq \chi_e(G)$.

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An *equitable coloring* of a graph G is a proper coloring $\mathcal{C}$ of G which is an assignment of colors to the vertices of G such that the numbers of vertices in any two color classes differ by at most one (see [10]). We denote this assignment by the function $\varphi_e : V(G) \to \mathcal{C}$ of the vertices of G with colors in $\mathcal{C}$. The *equitable chromatic number*, χ_e of a graph G, is the smallest number k such that G has an equitable coloring with k colors.

Some studies on equitable coloring parameters of certain graph classes have been conducted in [4, 12]. Motivated by the studies on different types of graph colorings, equitable coloring parameters and chromatic Zagreb indices [8], we define and discuss the concepts of equitable chromatic Zagreb indices and equitable chromatic irregularity indices of Mycielski of certain graph classes in this paper. Analogous to the definitions of Zagreb and irregularity indices of graphs (see [1, 5, 14, 15]), the notions of equitable chromatic Zagreb indices and equitable chromatic irregularity indices are defined as follows:

2. Equitable chromatic Zagreb and irregularity indices of graphs

Definition 2.1. Let G be a graph and let $\mathcal{C} = \{c_1, c_2, c_3, \ldots, c_\ell\}$ be an equitable coloring of G such that $\varphi_e(v_i) = c_s; 1 \le i \le n, 1 \le s \le \ell$. Then for $1 \le t \le \ell\,!$,

(i) The *first equitable chromatic Zagreb index* of G, denoted by $M_1^{\varphi_e t}(G)$, is defined as $M_1^{\varphi_e t}(G) = \sum\limits_{i=1}^{n} (\zeta(v_i))^2$.

(ii) The *second equitable chromatic Zagreb index* of G, denoted by $M_2^{\varphi_e t}(G)$, is defined as $M_2^{\varphi_e t}(G) = \sum\limits_{i=1}^{n-1} \sum\limits_{j=2}^{n} (\zeta(v_i) \cdot \zeta(v_j)),\ v_iv_j \in E(G)$.

(iii) The *equitable chromatic irregularity index* of G, denoted by $M_3^{\varphi_e t}(G)$, is defined as $M_3^{\varphi_e t}(G) = \sum\limits_{i=1}^{n-1} \sum\limits_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)|,\ v_iv_j \in E(G)$.

(iv) The *equitable chromatic total irregularity index* of G, denoted by $M_4^{\varphi_e t}(G)$, is defined as $M_4^{\varphi_e t}(G) = \frac{1}{2} \sum\limits_{i=1}^{n-1} \sum\limits_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)|,\ v_i, v_j \in V(G)$.

In view of the above notions, the minimum and maximum equitable chromatic Zagreb indices and the corresponding irregularity indices are defined as follows.

$$
\begin{aligned}
M_1^{\varphi_e^-}(G) &= \min\{M_1^{\varphi_e t}(G) : 1 \le t \le \ell!\},\\
M_1^{\varphi_e^+}(G) &= \max\{M_1^{\varphi_e t}(G) : 1 \le t \le \ell!\},\\
M_2^{\varphi_e^-}(G) &= \min\{M_2^{\varphi_e t}(G) : 1 \le t \le \ell!\},\\
M_2^{\varphi_e^+}(G) &= \max\{M_2^{\varphi_e t}(G) : 1 \le t \le \ell!\},\\
M_3^{\varphi_e^-}(G) &= \min\{M_3^{\varphi_e t}(G) : 1 \le t \le \ell!\},\\
M_3^{\varphi_e^+}(G) &= \max\{M_3^{\varphi_e t}(G) : 1 \le t \le \ell!\}\\
M_4^{\varphi_e^-}(G) &= \min\{M_4^{\varphi_e t}(G) : 1 \le t \le \ell!\},\\
M_4^{\varphi_e^+}(G) &= \max\{M_4^{\varphi_e t}(G) : 1 \le t \le \ell!\}.
\end{aligned}
$$

3. Equitable chromatic Zagreb index of Mycielskian of a graph

Motivated by the studies mentioned above, we study the equitable chromatic Zagreb indices and equitable chromatic irregularity indices of Mycielskian of certain fundamental graph classes in the following discussion.

Definition 3.2. [9] Let G be a graph with the vertex set $V(G) = \{v_1, \ldots, v_n\}$. The *Mycielski graph* or the *Mycielskian* of a graph G, denoted by $\mu(G)$, is the graph with vertex set $V(\mu(G)) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, w\}$ such that $v_iv_j \in E(\mu(G)) \iff v_iv_j \in E(G)$, $v_iu_j \in E(\mu(G)) \iff v_iv_j \in E(G)$ and $u_iw \in E(\mu(G))$ for all $i = 1, \ldots, n$.

For the ease of the notation, we denote the Mycielski graph of a graph G by $\breve{G}$.

Theorem 3.1. *For the Mycielskian of a path P_n, we have*

(i) $M_1^{\varphi_e^-}(\breve{P}_n) = \begin{cases} 15n+1; & \text{if } n \text{ is even} \\ 15n-1; & \text{if } n \text{ is odd;} \end{cases}$

(ii) $M_2^{\varphi_e^-}(\breve{P}_n) = \begin{cases} \frac{31n-24}{2}; & \text{if } n \text{ is even} \\ \frac{33n-27}{2}; & \text{if } n \text{ is odd;} \end{cases}$

(iii) $M_3^{\varphi_e^-}(\breve{P}_n) = \begin{cases} 6n-4; & \text{if } n \text{ is even} \\ \frac{9n-7}{2}; & \text{if } n \text{ is odd;} \end{cases}$

(iv) $M_4^{\varphi_e^-}(\breve{P}_n) = \begin{cases} \frac{5n^2+6n}{4}; & \text{if } n \text{ is even} \\ 2n^2+n-1; & \text{if } n \text{ is odd.} \end{cases}$

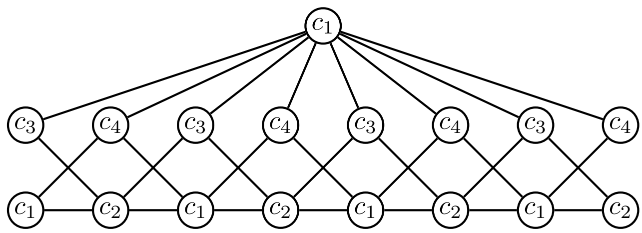

FIGURE 1. Equitable coloring of $\breve{P}_8$

Proof. Consider a path P_n with vertex set $\{v_1, v_2, \ldots, v_n\}$ and it's Mycielskian $\breve{P}_n$, with vertex set $\{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, w\}$. Then, $\breve{P}_n$ has $2n+1$ vertices and $4n-3$ edges.The vertices u_i and v_i may be called the *twin vertices* and the vertex w may be called the *root vertex*. We introduce here the notation η_{ts} to denote the number of edges with end points t and s respectively, where $t < s, 1 \leq t, s \leq \chi_e(\breve{P}_n)$. Now we proceed with the proof considering even and odd cases separately.

Case-1: Let n be even. It is clear that $\chi_e(\breve{P}_n) = 4$. For the easy flow of the proof we define four independent sets in $\breve{P}_n$ as $S_1 = \{w, v_1, v_3, \ldots, v_{n-1}\}, S_2 = \{v_2, v_4, \ldots, v_n\}, S_3 = \{u_1, u_3, \ldots, u_{n-1}\}$ and $S_4 = \{u_2, u_4, \ldots, u_n\}$. Now we color S_i with color c_i for $1 \leq i \leq 4$ to have the values $\theta(c_1) = \frac{n+2}{2}$, $\theta(c_2) = \theta(c_3) = \theta(c_4) = \frac{n}{2}$ and $\eta_{12} = \eta_{23} = n-1, \eta_{13} = \frac{n}{2}, \eta_{14} = \frac{3n-2}{2}$.

Part (i): From the definition of the first equitable chromatic Zagreb index the result follows as:

$$M_1^{\varphi_e^-}(\breve{P}_n) = \sum_{i=1}^{4} (\theta(c_i))i^2 = 1 + 30\frac{n}{2} = 15n + 1.$$

Part (ii): From the definition of the second equitable chromatic Zagreb indices the result follows as:

$$M_2^{\varphi_e^-}(\breve{P}_n) = \sum_{1 \leq t, s \leq \chi_e(\breve{P}_n)}^{t<s} ts\eta_{ts} = 8(n-1) + \frac{3n}{2} + 4\frac{3n-2}{2} = \frac{31n-24}{2}.$$

Part (iii): In this case, S_1 is colored with c_1, S_2 with c_3, S_3 with c_4 and S_4 with c_2. So we will have $\eta_{34} = \eta_{13} = n-1, \eta_{14} = \frac{n}{2}, \eta_{12} = \frac{3n-2}{2}$. Now the definition of the equitable chromatic irregularity indices of a graph, gives the result as follows:

$$M_3^{\varphi_e^-}(\breve{P}_n) = \sum_{i=1}^{n-1}\sum_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)| = 3(n-1) + \frac{3n}{2} + \frac{3n-2}{2} = 6n-4.$$

Part (iv): First we assign equitable coloring to the vertices as S_i is colored with c_i for $1 \leq i \leq 4$. In order to calculate the equitable chromatic total irregularity of $\breve{P}_n$, all the possible vertex pairs from $\breve{P}_n$ have to be considered and their possible color distances are determined. We observe that vertex pairs with same colors contribute nothing to the color distance and we discard such cases. The possibility of the vertex pairs which contribute to the color distance are calculated considering all vertex pairs. Now from the definition of the equitable chromatic total irregularity indices of a graph, the result follows as:

$$M_4^{\varphi_e^-}(\breve{P}_n) = \frac{1}{2}\sum_{u,v \in V(\breve{P}_n)} |\varphi_e(u) - \varphi_e(v)| = \frac{1}{2}\left(n^2 + 3n\left(\frac{n+2}{2}\right)\right) = \frac{5n^2+6n}{4}.$$

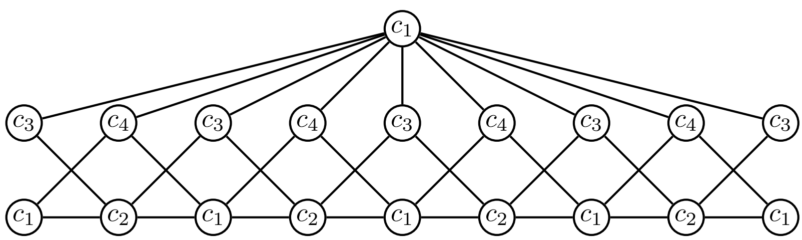

FIGURE 2. Equitable coloring of $\breve{P}_9$

Case-2: Let n be odd. It is clear that $\chi_e(\breve{P}_n) = 4$. Here we define 4 independent sets in $\breve{P}_n$ as $S_1 = \{v_1, v_3, \ldots, v_n\}, S_2 = \{w, v_2, v_4, \ldots, v_{n-1}\}, S_3 = \{u_1, u_3, \ldots, u_n\}$ and $S_4 = \{u_2, u_4, \ldots, u_{n-1}\}$. Here, S_1 is colored with c_2, S_2 with c_1, S_3 with c_3 and S_4 with c_4 to have the values $\theta(c_1) = \theta(c_2) = \theta(c_3) = \frac{n+1}{2}$, $\theta(c_4) = \frac{n-1}{2}$ and $\eta_{12} = \eta_{24} = n-1, \eta_{13} = \frac{3n-1}{2}, \eta_{14} = \frac{n-1}{2}$.

Part (i): From the definition of the first equitable chromatic Zagreb indices the result follows as:

$$M_1^{\varphi_e^-}(\breve{P}_n) = \sum_{i=1}^{4}(\theta(c_i))i^2 = 7(n+1) + 8(n-1) = 15n-1.$$

Part (ii): From the definition of the second equitable chromatic Zagreb indices the result follows as:

$$M_2^{\varphi_e^-}(\breve{P}_n) = \sum_{1 \leq t,s \leq \chi_e(\breve{P}_n)}^{t<s} ts\eta_{ts} = 12(n-1) + 3\frac{3n-1}{2} = \frac{33n-27}{2}.$$

Part (iii): Here S_1 is colored with c_3, S_2 with c_2, S_3 with c_1 and S_4 with c_4. So we will have $\eta_{23} = \eta_{34} = n-1, \eta_{24} = \frac{n-1}{2}, \eta_{12} = \frac{3n-1}{2}$. Now the definition of the equitable chromatic irregularity indices of a graph, gives the result as follows:

$$M_3^{\varphi_e^-}(\breve{P}_n) = \sum_{i=1}^{n-1}\sum_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)| = 3(n-1) + \frac{3n-1}{2} = \frac{9n-7}{2}.$$

Part (iv): In this case, S_1 is colored with c_2, S_2 with c_1, S_3 with c_3 and S_4 with c_4 to have the values $\theta(c_1) = \theta(c_2) = \theta(c_3) = \frac{n+1}{2}$, $\theta(c_4) = \frac{n-1}{2}$ and $\eta_{12} = \eta_{24} = n-1, \eta_{13} = \frac{3n-1}{2}, \eta_{14} = \frac{n-1}{2}$. Now from the definition of the equitable chromatic total irregularity indices of a graph, the result follows as:

$$M_4^{\varphi_e^-}(\breve{P}_n) = \frac{1}{2}\sum_{u,v\in V(\breve{P}_n)} |\varphi_e(u) - \varphi_e(v)| = \frac{1}{2}((n+1)^2 + 3(n^2-1)) = 2n^2 + n - 1.$$

□

Theorem 3.2. *For the Mycielskian of a path P_n, we have*

(i) $M_1^{\varphi_e^+}(\breve{P}_n) = \begin{cases} 15n+16; & \text{if } n \text{ is even} \\ 15n; & \text{if } n \text{ is odd}; \end{cases}$

(ii) $M_2^{\varphi_e^+}(\breve{P}_n) = \begin{cases} 30n-22; & \text{if } n \text{ is even} \\ \frac{55n-31}{2}; & \text{if } n \text{ is odd}; \end{cases}$

(iii) $M_3^{\varphi_e^+}(\breve{P}_n) = \begin{cases} \frac{15n-10}{2}; & \text{if } n \text{ is even} \\ \frac{13(n-1)}{2}; & \text{if } n \text{ is odd}; \end{cases}$

(iv) $M_4^{\varphi_e^+}(\breve{P}_n) = \begin{cases} \frac{7n^2+6n}{4}; & \text{if } n \text{ is even} \\ 2n^2+n-1; & \text{if } n \text{ is odd}. \end{cases}$

Proof. Here the proof is almost similar to that of the proof of Theorem 3.1 and so we proceed with the proof considering even and odd cases separately.

Case-1: Let n be even. It is clear that $\chi_e(\breve{P}_n) = 4$. We define 4 independent sets in $\breve{P}_n$ as $S_1 = \{w, v_1, v_3, \ldots, v_{n-1}\}, S_2 = \{v_2, v_4, \ldots, v_n\}, S_3 = \{u_1, u_3, \ldots, u_{n-1}\}$ and $S_4 = \{u_2, u_4, \ldots, u_n\}$. Here, S_1 is colored with c_4, S_2 with c_2, S_3 with c_1 and S_4 with c_3 to have the values $\theta(c_4) = \frac{n+2}{2}$, $\theta(c_3) = \theta(c_2) = \theta(c_1) = \frac{n}{2}$ and $\eta_{12} = \eta_{24} = n-1, \eta_{14} = \frac{n}{2}, \eta_{34} = \frac{3n-2}{2}$.

Part (i): From the definition of the first equitable chromatic Zagreb indices the result follows as:

$$M_1^{\varphi_e^+}(\breve{P}_n) = \sum_{i=1}^{4}(\theta(c_i))i^2 = 7n + 16\frac{n+2}{2} = 15n + 16.$$

Part (ii): From the definition of the second equitable chromatic Zagreb indices the result follows as:

$$M_2^{\varphi_e^+}(\breve{P}_n) = \sum_{1\le t,s\le \chi_e(\breve{P}_n)}^{t<s} ts\eta_{ts} = 10(n-1) + \frac{4n}{2} + 12\frac{3n-2}{2} = 30n - 22.$$

Part (iii): Here S_1 is colored with c_4, S_2 with c_3, S_3 with c_1 and S_4 with c_2. So we will have $\eta_{34} = \eta_{13} = n-1, \eta_{14} = \frac{n}{2}, \eta_{12} = \frac{3n-2}{2}$. Now the definition of the equitable chromatic irregularity indices of a graph, gives the result as follows:

$$M_3^{\varphi_e^-}(\breve{P}_n) = \sum_{i=1}^{n-1}\sum_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)| = 3(n-1) + \frac{3n}{2} + 4\frac{3n-2}{2} = \frac{15n-10}{2}.$$

Part (iv): First we assign equitable coloring to the vertices as described above for part one and two. Now from the definition of the equitable chromatic total irregularity indices of a graph, the result follows as:

$$M_4^{\varphi_e^+}(\breve{P}_n) = \frac{1}{2}\sum_{u,v\in V(\breve{P}_n)} |\varphi_e(u) - \varphi_e(v)| = \frac{1}{2}(2n^2 + 3\frac{n^2+2n}{2}) = \frac{7n^2+6n}{4}$$

Case-2: Let n be odd. It is clear that $\chi_e(\breve{P}_n) = 4$. Here we define 4 independent sets in $\breve{P}_n$ as $S_1 = \{v_1, v_3, \dots, v_n\}, S_2 = \{w, v_2, v_4, \dots, v_{n-1}\}, S_3 = \{u_1, u_3, \dots, u_n\}$, and $S_4 = \{u_2, u_4, \dots, u_{n-1}\}$. Here, S_1 is colored with c_2, S_2 with c_1, S_3 with c_3 and S_4 with c_4 to have the values $\theta(c_4) = \theta(c_2) = \theta(c_3) = \frac{n+1}{2}$, $\theta(c_1) = \frac{n-1}{2}$ and $\eta_{12} = \eta_{23} = n-1, \eta_{34} = \frac{3n-1}{2}, \eta_{13} = \frac{n-1}{2}$.

Part (i): From the definition of the first equitable chromatic Zagreb indices the result follows as:

$$M_1^{\varphi_e^+}(\breve{P}_n) = \sum_{i=1}^{4}(\theta(c_i))i^2 = \frac{29n+1}{2} + \frac{n-1}{2} = 15n.$$

Part (ii): From the definition of the second equitable chromatic Zagreb indices the result follows as:

$$M_2^{\varphi_e^+}(\breve{P}_n) = \sum_{1\leq t,s\leq \chi_e(\breve{P}_n)}^{t<s} ts\eta_{ts} = \frac{19(n-1)}{2}) + 12\frac{3n-1}{2} = \frac{55n-31}{2}$$

Part (iii): Here S_1 is colored with c_3, S_2 with c_2, S_3 with c_4 and S_4 with c_1. So we will have $\eta_{23} = \eta_{13} = n-1, \eta_{12} = \frac{n-1}{2}, \eta_{24} = \frac{3n-1}{2}$. Now the definition of the equitable chromatic irregularity indices of a graph, gives the result as follows:

$$M_3^{\varphi_e^+}(\breve{P}_n) = \sum_{i=1}^{n-1}\sum_{j=2}^{n}|\zeta(v_i) - \zeta(v_j)| = 3(n-1) + \frac{7(n-1)}{2} = \frac{13(n-1)}{2}.$$

Part (iv): Here the proof of the result is as same as in theorem 3.2 and from the definition of the equitable chromatic total irregularity indices of a graph, the result follows as:

$$M_4^{\varphi_e^+}(\breve{P}_n) = \frac{1}{2}\sum_{u,v\in V(\breve{P}_n)} |\varphi_e(u) - \varphi_e(v)| = \frac{1}{2}((n+1)^2 + 3(n^2-1)) = 2n^2 + n - 1.$$

$\square$

Theorem 3.3. *For the Mycielskian of a cycle C_n, we have*

(i) $M_1^{\varphi_e^-}(\breve{C}_n) = \begin{cases} 15n+1; & \textit{if } n \textit{ is even} \\ 15n-1; & \textit{if } n \textit{ is odd}; \end{cases}$

(ii) $M_2^{\varphi_e^-}(\breve{C}_n) = \begin{cases} \frac{31n}{2}; & \textit{if } n \textit{ is even} \\ \frac{31n+29}{2}; & \textit{if } n \textit{ is odd}; \end{cases}$

(iii) $M_3^{\varphi_e^-}(\breve{C}_n) = \begin{cases} \frac{15n}{2}; & \textit{if } n \textit{ is even} \\ \frac{15n-13}{2}; & \textit{if } n \textit{ is odd}; \end{cases}$

(iv) $M_4^{\varphi_e^-}(\breve{C}_n) = \begin{cases} \frac{5n^2+6n}{4}; & \textit{if } n \textit{ is even} \\ \frac{5n^2+4n-1}{4}; & \textit{if } n \textit{ is odd}. \end{cases}$

Proof. Consider a cycle C_n with vertex set $\{v_1, v_2, \dots, v_n\}$ and it's Mycielskian $\breve{C}_n$, with vertex set $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n, w\}$. Then, $\breve{C}_n$ has $2n+1$ vertices and $4n$ edges.The vertices u_i and v_i may be called the *twin vertices* and the vertex w may be called the *root vertex*. Now we proceed with the proof considering even and odd cases separately.

Case-1: Let n be even. It is clear that $\chi_e(\breve{C}_n) = 4$. For the easy flow of the proof we define four independent sets in $\breve{P}_n$ as $S_1 = \{w, v_1, v_3, \dots, v_{n-1}\}, S_2 = \{v_2, v_4, \dots, v_n\}, S_3 = \{u_1, u_3, \dots, u_{n-1}\}$, and $S_4 = \{u_2, u_4, \dots, u_n\}$. Now we color S_i with color c_i for $1 \leq i \leq 4$ to have the values $\theta(c_1) = \frac{n+2}{2}$, $\theta(c_2) = \theta(c_3) = \theta(c_4) = \frac{n}{2}$ and $\eta_{12} = \eta_{23} = n, \eta_{13} = \frac{n}{2}, \eta_{14} = \frac{3n}{2}$.

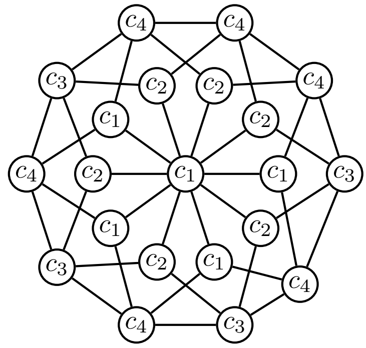

FIGURE 3. An equitable coloring of $\breve{C}_{10}$.

Part (i): From the definition of the first equitable chromatic Zagreb indices the result follows as:

$$M_1^{\varphi_e^-}(\breve{C}_n) = \sum_{i=1}^{4}(\theta(c_i))i^2 = \frac{n+2}{2} + 29\frac{n}{2} = 15n + 1.$$

Part (ii): From the definition of the second equitable chromatic Zagreb indices the result follows as:

$$M_2^{\varphi_e^-}(\breve{C}_n) = \sum_{1\le t,s\le \chi_e(\breve{C}_n)}^{t<s} ts\eta_{ts} = 14n + \frac{3n}{2} = \frac{31n}{2}.$$

Part (iii): From the definition of the equitable chromatic irregularity indices of a graph, gives the result as follows:

$$M_3^{\varphi_e^-}(\breve{C}_n) = \sum_{i=1}^{n-1}\sum_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)| = 2n + \frac{11n}{2} = \frac{15n}{2}.$$

Part (iv): First we assign equitable coloring to the vertices as S_i is colored with c_i for $1 \le i \le 4$. In order to calculate the equitable chromatic total irregularity of $\breve{C}_n$, all the possible vertex pairs from $\breve{C}_n$ have to be considered and their possible color distances are determined. We observe that vertex pairs with same colors contribute nothing to the color distance and we discard such cases. The possibility of the vertex pairs which contribute to the color distance are calculated considering all vertex pairs. Now from the definition of the equitable chromatic total irregularity indices of a graph, the result follows as:

$$M_4^{\varphi_e^-}(\breve{C}_n) = \frac{1}{2}\sum_{u,v\in V(\breve{C}_n)} |\varphi_e(u) - \varphi_e(v)| = \frac{1}{2}\left(n^2 + 3n\cdot\frac{n+2}{2}\right) = \frac{5n^2+6n}{4}.$$

Case-2: Let n be odd. Here also $\chi_e(\breve{C}_n) = 4$. Here we define 4 independent sets in $\breve{C}_n$ as $S_1 = \{v_1, v_3, \ldots, v_{n-2}, w\}, S_2 = \{v_2, v_4, \ldots, v_{n-3}, v_n, u_{n-3}\}, S_3 = \{u_1, u_3, \ldots, u_{n-4}, u_{n-1}, v_{n-1}\}$ and $S_4 = \{u_2, u_4, \ldots, u_{n-5}, u_{n-2}, u_n\}$. Here, S_1 is colored with c_1, S_2 with c_2, S_3 with c_3 and S_4 with c_4 to have the values $\theta(c_1) = \theta(c_2) = \theta(c_3) = \frac{n+1}{2}$, $\theta(c_4) = \frac{n-1}{2}$ and $\eta_{12} = n+1, \eta_{13} = \frac{n+3}{2}, \eta_{14} = \frac{3(n-3)}{2}, \eta_{23} = n-1, \eta_{24} = 1, \eta_{34} = 2.$

Part (i): From the definition of the first equitable chromatic Zagreb indices the result follows as:

$$M_1^{\varphi_e^-}(\breve{C}_n) = \sum_{i=1}^{4}(\theta(c_i))i^2 = 7(n+1) + 8(n-1) = 15n - 1.$$

Part (ii): From the definition of the second equitable chromatic Zagreb indices the result follows as:

$$M_2^{\varphi_e^-}(\breve{C}_n) = \sum_{1\le t,s\le \chi_e(\breve{C}_n)}^{t<s} ts\eta_{ts} = 2(n+1) + 3\frac{n+3}{2} + 6(n-3) + 6(n-1) + 32 = \frac{31n+29}{2}$$

Part (iii): From the definition of the equitable chromatic irregularity indices of a graph, gives the result as follows:

$$M_3^{\varphi_e^-}(\breve{C}_n) = \sum_{i=1}^{n-1}\sum_{j=2}^{n} |\zeta(v_i) - \zeta(v_j)| = (n+1) + (n+3) + (n-1) + \frac{9(n-3)}{2} + 4 = \frac{15n-13}{2}$$

Part (iv): Now from the definition of the equitable chromatic total irregularity indices of a graph, the result follows as:

$$M_4^{\varphi_e^-}(\breve{C}_n) = \frac{1}{2}\sum_{u,v\in V(\breve{C}_n)} |\varphi_e(u) - \varphi_e(v)| = \frac{1}{2}((n+1)^2 + 3(n^2-1)) = \frac{5n^2+4n-1}{4}$$

□

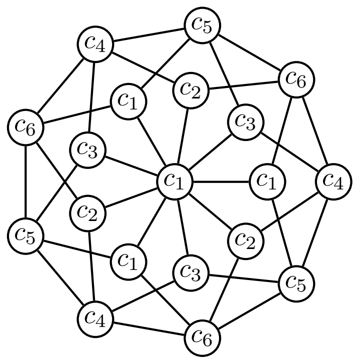

FIGURE 4. An equitable coloring of $\breve{C}_9$.

Theorem 3.4. *For the Mycielskian of a cycle C_n, we have*

(i) $M_1^{\varphi_e^+}(\breve{C}_n) = \begin{cases} 15n+16; & \text{if } n \text{ is even} \\ \frac{30n+28}{2}; & \text{if } n \text{ is odd;} \end{cases}$

(ii) $M_2^{\varphi_e^+}(\breve{C}_n) = \begin{cases} 30n; & \text{if } n \text{ is even} \\ 28n+7; & \text{if } n \text{ is odd;} \end{cases}$

(iii) $M_3^{\varphi_e^+}(\breve{C}_n) = \begin{cases} \frac{15n}{2}; & \text{if } n \text{ is even} \\ \frac{15n-9}{2}; & \text{if } n \text{ is odd;} \end{cases}$

(iv) $M_4^{\varphi_e^+}(\breve{C}_n) = \begin{cases} \frac{5n^2+6n}{4}; & \text{if } n \text{ is even} \\ \frac{5n^2+4n-1}{4}; & \text{if } n \text{ is odd.} \end{cases}$

Proof. Proof of the theorem is almost similar to that of the proof of Theorem 3.2. □

4. CONCLUSION

In this paper, we determined four important chromatic topological indices, related to equitable coloring of certain graph classes derived from paths and cycles. These parameters defined for graph coloring problems can be used in various areas like project management, communication networks, optimization problems etc. The concepts of equitable chromatic parameters can be utilized in certain practical and industrial problems like resource allocation, resource smoothing, inventory management, service and distribution systems etc.

The equitable chromatic topological indices of several other graph classes are yet to be studied. Further investigations on the other topological indices corresponding to the equitable coloring of many other standard graphs seem to be promising open problems. Studies on the graph operations in correspondence to different types of edge colorings, map colorings, total colorings etc. of graphs also offer much for future studies. Also, a comparative study on chromatic Zagreb indices and irregularity indices of graph classes and their operations will be interesting.

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