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The Cartan connection: sketches for a portrait of Kentaro Yano

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ABSTRACT. We are describing the historical context in which Kentaro Yano prepared his doctoral dissertation under Élie Cartan's coordination, and how this work was published in Romania, with *Analele Ştiinţifice ale Universităţii "Al. I. Cuza"*. We describe some of the many encounters made possible by Élie Cartan's extraordinary creative contributions, which lead to a series of important collaborations, some of them of lasting impact until today.

In Kentaro Yano's biography [29] the information on his doctoral dissertation is not included, and no information on its actual publication is given. According to Leopold Verstraelen [32], Radu Roşca recalled that there was a pre-WW II connection between Kentaro Yano's doctoral thesis and the Romanian academic environment. Since several resources about Kentaro Yano's biography do not incorporate this information, we hereby complement this important reference, which should be viewed in the historical context pertaining to the period 1936-1939. The few elements we hereby describe have all to do with the extraordinary impact of Élie Cartan's work and inspiring presence in the mathematical environment in the 1930s; in a certain way, the present note could be viewed as a complement to the comprehensive monograph [2], to support the assertion that Élie Cartan's work raised major attention and brought together mathematicians from many traditions, contributing essentially to international communication and collaboration.

In June 1938, Kentaro Yano defended at the University of Paris his doctoral dissertation titled *Les espaces à connexion projective et la géométrie projective des "path"*, with Élie Cartan as doctoral adviser and G. Valiron and G. Darmois as committee members [34]. It was the culmination of two years of intensive study under Cartan's guidance. The actual thesis is published as [35]. The fact that this is the actual doctoral thesis is also confirmed in [28], at p. xxxvi.

As the articles bear the mark of the original printing outlet, we can see that the original work was printed with *Presa Bună*, a facility established by the Catholic Diocese in Iaşi in October 1926, where at that time existed the technical ability to print mathematical fonts.

According to his autobiography, Kentaro Yano left Paris in 1938, and in Japan defended a second doctoral thesis [36]. It is quite possible that the administrative environment in Japan at that time made this choice necessary.

Élie Cartan had many connections in the Romanian mathematical circles, starting with Gheorghe Ţiţeica (1873-1939), who was [1], like Cartan himself, a former doctoral student of Gaston Darboux (1872-1917). Ţiţeica and Cartan have not been classmates, but it is very likely they met in Paris before 1899. According to [2], p.6, Élie Cartan graduated from l'École Normale Superieure in 1891, then served in the army, and subsequently worked on his doctoral thesis for two years, 1892-1894, in Paris. Between 1894 and 1896, Cartan was a lecturer at the University of Montpellier, and between 1896 and 1903 was a

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FIGURE 1. In this image, Tadashi Nagano (left, a former doctoral student of Kentaro Yano), Kentaro Yano (center), and Katsumi Nomizu (right). Photo from Tadashi Nagano's family archive, reprinted with Reiko Nagano's permission.

lecturer at the Faculty of Sciences of the University of Lyon [2], p.7. Ţiţeica was in Paris between the fall of 1896 and the summer of 1899, when he returned to Bucharest [1]. Cartan met with Ţiţeica several times, at the International Congresses of Mathematicians in 1920 (Strasbourg), 1924 (Bologna), 1932 (Zürich), and 1936 (Oslo). Over the years, Cartan had an extended communications with several Romanian mathematicians, out of which there is published his correspondence with Ţiţeica, Alexandru Pantazi (1873-1939), and Gheorghe Vrănceanu (1900-1979) [9], fact mentioned in [2], p. 28. Akivis and Rosenfeld describe in [2], p. 28: "In April and May of 1931 Cartan made a trip to Romania and Poland. In Romania he delivered a series of lectures in Cluj, Bucharest, Iaşi (Yassy), and Cernăuţi (Chernovcy, after WW II in U.S.S.R., [and thereafter in Ukraine]). In the same year, Cartan was elected an honorary member of the Romanian Academy of Sciences in Bucharest."

There is an outstanding testimony of the encounter in Bucharest, where Cartan's visit coincided with a meeting of the local mathematical society, Societatea de Matematică (established in 1909, today continued by a society in partnership with the American Mathematican Society). The witness of Cartan's presence in Bucharest was nobody else but Dan Barbilian (1895-1961), see e.g.[7], who writes in a note published only after his death the following description, [3], p. 171.

> In the meeting of the Society of Mathematics from May 1931, Élie Cartan presented a talk on the geometry of the isotropic curves and manifolds. At the end, Gh. Ţiţeica referred to his "5-lei problem", for the first time called so in public, and about its first solution I gave, but with some formal simplifications brought to it in the meantime.
>
> I was a sort of ad-hoc secretary of that particular meeting. I just found recently the notes I took at that time. Ţiţeica, this man full of intuition,

> asked the question whether on the sphere or on other surfaces the "5-lei configuration" holds true (that is, of equiradial geodesic circles).
>
> However, Ţiţeica's question makes sense only on surfaces with constant curvature. It is there, on a very small patch on that surface, the geodesic circles (the locus of the points equidistant *on the surface* [Barbilian's underlining] from a fixed point; the path minimizer between two given points, thus the arc of geodesic passing by there) cuts two by two and the construction is unequivocal.
>
> Thus, the construction makes sense only on the so-called symmetrizable surfaces introduced in Geometry by Élie Cartan himself.

Based on this first-hand testimonial, we see that the conversations that took place around Cartan's talk had to do with symmetric spaces, as Cartan was reflecting at that time on this new concept [8]. It is possible that in the audience were, besides Ţiţeica, Dimitrie Pompeiu, Simeon Stoilow, Gheorghe Vrănceanu, and other first class mathematicians. The "5-lei problem" is what we call today the Tzitzeica-Johnson configuration, see e.g. [5]. What is particularly remarkable is that in this encounter we see ideas pertaining today to comparison geometry, and that this conversation took place before Topogonov's theorem was established [31]. For the origins of the important idea of triangle comparison, see the thorough historical account in [30], and for the use of Tzitzeica-Johnson configuration as criterion, see [6].

Cartan's visit in Romania was full of substantive academic encounters. In Iaşi, the mathematical community benefitted from the expertise of the excellent geometer Alexandru Myller, and it is very likely that Cartan was received as warmly as in Bucharest. It is therefore conceivable that Kentaro Yano's 1938 doctoral dissertation was welcomed with great interest by an academic environment fully connected to Cartan's mathematical ideas. Thus, the publication of [35] was yet one more time a reaffirmation of an important academic connection that could be traced four decades back, since the 1890s.

FIGURE 2. Kentaro Yano and Bang-Yen Chen, in East Lansing, MI, on May 14, 1972. Photo reprinted with the generous permission of Dr. Bang-Yen Chen.

After 1938, Kentaro Yano worked at the Tokyo Institute of Technology. As it is pointed out in [29], "He remained on the staff at the Institute for the whole of his career but made many trips abroad and spent long periods in other universities." After WW II, Kentaro Yano became widely known, mainly due to his monographs [37, 38, 39], to mention here just a few of his very influential works. As visiting professor to Princeton, he wrote with Salomon Bochner [4], a reference still of interest today. Besides this collaboration, Kentaro Yano developed very successful collaborations with other authors, most notably with Shigeru Ishihara and Masahiro Kon.

Kentaro Yano's academic prestige and influence was extremely important. To cite one notable example, Sh. Kobayashi mentions in his foreword of the work [26], published first in Japanese in 1977, and translated into English only recently, that: "I would like to thank my former teacher, Professor Kentano Yano, who suggested me to write this book and has advised me over many years."

An outstanding research relationship developed between Kentaro Yano and Bang-Yen Chen, who describes in [10], in stellar terms their very fruitful collaboration.

> I met Professor Kentaro Yano for the first time in late 1970 at Michigan State University while he was a visiting professor under the invitation of Professors D. E. Blair and G. D. Ludden. Since then he had visited MSU as visiting professor for a few times until 1973; about one month for each visit. During his visits at MSU, we had many discussions on geometry of conformally flat manifolds and submanifolds, among others. Usually, we discussed after lunch until 5 pm almost every week day. Hence many of our joint articles done during that period are related to conformally flat manifolds and submanifolds [15, 16, 17, 18, 40]. From his visits at MSU we also published several articles on integral formulas and pseudo-umbilical or quasi-umbilical submanifolds (see [11, 12, 13, 14, 20, 21, 22, 23]). Later, we published two joint articles [24, 25] on different subjects via communications by air mails.
>
> From his visits at MSU, I learned very much from him. Most importantly, I tried to learn from his skill as well as from his philosophy on research. Consequently, Professor Yano had very important impact on my research as well.

In many ways, this was over many decades a very seminal heritage coming from a common academic tradition, doubled by many direct encounters and many personal conversations.

The journal in Iaşi published over the years several important papers; most notable is the publication in 1965 of T.J. Willmore's paper [33] in which the famous Willmore conjecture was first stated. Since then, the conjecture was solved in 2014 in [27].

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