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Supra soft b-connectedness II: Some types of supra soft b-connectedness

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ABSTRACT. This work is divided into two parts. In this second part, we introduce more properties of the notion of supra soft b-connectedness considered in the first part [Abd El-latif, A. M., *Supra soft b-connectedness I: Supra soft b-irresoluteness and separateness*, Creat. Math. Inform., **25** (2016), No. 2, 127–134]. Further, we introduce some types of supra soft connectedness in terms of supra b-open soft sets namely, supra soft locally b-connected, supra soft b-hyperconnected and study some of their properties.

1. INTRODUCTION

In 1983, Mashhour et al. [15] introduced the supra topological spaces, not only, as a generalization to the class of topological spaces, but also, these spaces were easier in the application as shown in [5]. In 2001, Popa et al. [19] generalized the supra topological spaces to the minimal spaces and generalized spaces as a new wider classes. In 2001, El-Sheikh [8] succeeded to use the fuzzy supra topology to study some topological properties to the fuzzy bitopological spaces. In 2007, Arpad Szaz [20] succeeded to introduce an application on the minimal spaces and generalized spaces. In 1987, Abd El-Monsef et al. [2] introduced the fuzzy supra topological spaces.

The concept of soft sets was first introduced by Molodtsov [16] in 1999 as a general mathematical tool for dealing with uncertain objects, in order to solve complicated problems in economics, engineering and the like. In [16, 17], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.

After presentation of the operations of soft sets [14], the properties and applications of soft set theory have been studied increasingly [4, 12, 17, 18]. Recently, in 2011, Shabir and Naz [21] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. It got some stability only after the introduction of soft topology [21] in 2011. In [10], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, $\alpha-$open soft and β-open soft and investigated their properties in detail. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [7]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of b-open soft sets was initiated by El-sheikh and Abd El-latif [6]. An applications on b-open soft sets were introduced in [3, 9]. The notion of supra b-open soft sets was initiated by Abd El-latif et al. [3].

This work is divided into two parts. In the first part [1], the concept of supra b-irresolute soft functions is introduced, as a generalization to the supra b-continuous soft functions and several properties are investigated. Further, we have introduced the notion of supra

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soft b-connectedness and gave the basic definitions and theorems about it. Finally, we showed that, the surjective supra b-irresolute soft image of supra soft b-connected spaces is also supra soft b-connected.

In this part, we introduce more properties of the notion of supra soft b-connectedness [1]. Further, we introduce some types of supra soft connectedness in terms of supra b-open soft sets namely, supra soft locally b-connected, supra soft b-hyperconnected and study some of their properties, in addition to the relation between them.

2. Preliminaries

In this section, we present the basic definitions and results of supra soft set theory which may found in earlier studies [3, 7].

Definition 2.1. [7] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq SS(X)_E$ is called supra soft topology on X with a fixed set E if

- **(1):** $\tilde{X}, \tilde{\varphi} \in \mu$,
- **(2):** the union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called supra soft topological space (or supra soft spaces) over X.

Definition 2.2. [3] Let (X, μ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then, (F, E) is called a supra b-open soft set if $(F, E) \tilde{\subseteq} cl^s(int^s(F, E)) \tilde{\cup} int^s(cl^s(F, E))$. The complement of a supra $b-$open soft set is a supra $b-$closed soft set. The set of all supra $b-$open soft sets is denoted by $SBOS(X, \mu, E)$, or $SBOS_E(X)$ and the set of all supra $b-$closed soft sets is denoted by $SBCS(X, \mu, E)$, or $SBCS_E(X)$.

Definition 2.3. [1] Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ is called supra b-irresolute soft if $f_{pu}^{-1}(F, B) \in SBOS_A(X)$ for each $(F, B) \in SBOS_B(X)$.

Definition 2.4. [1] Two non-null soft sets G_E and H_E of supra soft topological space (X, μ, E) are said to be supra soft b-separated sets if $G_E \tilde{\cap} bScl^s(H_E) = \tilde{\varphi}$ and $bScl^s(G_E) \tilde{\cap} (H, E) = \tilde{\varphi}$.

Definition 2.5. [1] Let (X, μ, E) be a supra soft topological space. A supra soft b-separation of $\tilde{X}$ is a pair of non-null proper supra b-open soft sets in μ such that $(F, E) \tilde{\cap} (G, E) = \tilde{\varphi}$ and $\tilde{X} = (F, E) \tilde{\cup} (G, E)$.

Definition 2.6. [1]A supra soft topological space (X, μ, E) is said to be a b-soft connected if and only if there is no supra soft b-separations of $\tilde{X}$. If (X, μ, E) has such supra soft b-separations, then (X, μ, E) is said to be a supra soft b-disconnected.

3. On supra soft b-connectedness

In this section, we introduce more properties of the notion of supra soft b-connectedness [1]. Further, we introduce some types of supra soft b-connectedness and study relation between them.

Definition 3.7. A soft subset F_E of a supra soft topological space (X, μ, E) is supra soft b-connected, if it is supra soft b-connected as a soft subspace. In other words, a soft subset F_E of a supra soft topological space (X, μ, E) is said to be a supra soft b-connected relative to $\tilde{X_E}$ if there is not exist two supra soft b-separated subsets H_E and G_E relative to $\tilde{X_E}$ and $F_E = H_E \tilde{\cup} G_E$. Otherwise, F_E is said to be a supra soft b-disconnected.

Remark 3.1. Each supra soft disconnected set is supra soft b-disconnected.

Corollary 3.1. Let (X, τ_1, E) and (X, τ_2, E) be soft topological spaces and μ_1 , μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively such that $\mu_2 \subseteq \mu_1$. If μ_1 is supra soft b-connected, then μ_2 is supra soft b-connected.

Proof. It is obvious. □

Definition 3.8. Let (X, τ, E) be a soft topological space, μ_1 be an associated supra soft topologies with τ_1 and $(Z, E)\tilde{\subseteq}\tilde{X}$ with $x_\alpha \in (Z, E)$. Then, the supra soft b-component of (Z, E) w.r.t. x_α is the maximal of all supra soft b-connected subspaces of (Z, μ_Z, E) containing x_α and denoted by $\tilde{SC}^s_b[(Z, E), x_\alpha]$ or $\tilde{SC}^s_b(Z_E, x_\alpha)$ *for short*, i.e

$$\tilde{SC}^s_b(Z_E, x_\alpha) = \tilde{\cup}\{Y_E\tilde{\subseteq}Z_E : x_\alpha \in Y_E,\ Y_E \ is\ soft\ \mathrm{b}{-}connected\}.$$

Corollary 3.2. The supra soft topological space (X, μ, E) is supra soft b-connected if and only if it is a supra soft b-component on $\tilde{X}$.

Proof. It is clear. □

Theorem 3.1. *Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Then,*

- **(1):** *Each soft point $x_\alpha \in \tilde{X}$ is contained in exactly one supra soft b-component of $\tilde{X}$.*
- **(2):** *Any two supra soft b-components w.r.t. two different soft points of x_α are either disjoint or identical.*

Proof. Obvious. □

Theorem 3.2. *If the non-null soft sets G_E and H_E of a soft topological space (X, τ, E) are supra soft b-separated, then ($G_E\tilde{\cup}H_E$) is supra soft b-disconnected.*

Proof. Let G_E and H_E be non-null supra soft b-separated sets, then there exist supra b-open soft sets U_E and V_E such that $G_E\tilde{\subseteq}U_E$, $H_E\tilde{\subseteq}V_E$ and $G_E\tilde{\cap}V_E = \tilde{\varphi}$, $H_E\tilde{\cap}U_E = \tilde{\varphi}$ from [Theorem 4.3, [1]]. Hence, $(G_E\tilde{\cup}H_E)\tilde{\cap}U_E = G_E$ and $(G_E\tilde{\cup}H_E)\tilde{\cap}V_E = H_E$. Consequently, $G_E\tilde{\cup}H_E$ is supra soft b-disconnected. □

Theorem 3.3. *$\tilde{X}$ is supra soft b-connected if and only if every non-null proper subset has a non-null supra soft b-boundary.*

Proof. **Necessity:** Let $\tilde{X}$ be supra soft b-disconnected, then $\tilde{X}$ has a proper supra b-clopen soft set F_E. Then, $bScl^s(F_E) = F_E = bSint^s(F_E) = \tilde{X_E} - bScl^s(\tilde{X_E} - F_E)$. Therefore, $b - Sbd(F_E) = bScl^s(F_E)\tilde{\cap}bScl^s(\tilde{X_E} - F_E) = \tilde{\varphi}$. Therefore, F_E has an empty soft b-boundary.

Sufficient: suppose that a non-null proper soft subset F_E has an empty soft b-boundary. Then, $b - Sbd(F_E) = bScl^s(F_E\tilde{\cap}bScl^s(\tilde{X_E} - F_E) = \tilde{\varphi}$. Consequently, $bScl^s(F_E\tilde{\subseteq}\tilde{X_E} - bScl^s(\tilde{X_E} - F_E) = bSint^s(F_E)$, and thus $F_E\tilde{\subseteq}bScl^s(F_E\tilde{\subseteq}bSint^s(F_E)\tilde{\subseteq}F_E$. Thus, F_E is a proper supra b-clopen soft set and consequently, (X, μ, E) is supra soft b-connected.

□

Theorem 3.4. *Let (Z, μ_Z, E) be a supra soft subspace of a supra soft topological space(X, μ, E) and $F_{1E}, F_{2E}\tilde{\subseteq}(Z, E)\tilde{\subseteq}\tilde{X}$. Then, F_{1E}, F_{2E} are supra soft b-separated on μ_Z if and only if F_{1E}, F_{2E} are supra soft b-separated on μ.*

Proof. Suppose that F_{1E}, F_{2E} are supra soft b-separated on $\mu_Z \Leftrightarrow bScl^s_{\mu_Z}F_{1E}\tilde{\cap}F_{2E} = \tilde{\varphi}$ and $F_{1E}\tilde{\cap}bScl^s_{\mu_Z}F_{2E} = \tilde{\varphi} \Leftrightarrow [bScl^s_\mu F_{1E}\tilde{\cap}(Z, E)]\tilde{\cap}F_{2E} = bScl^s_\mu F_{1E}\tilde{\cap}F_{2E} = \tilde{\varphi}$ and $[bScl^s_\mu F_{2E}\tilde{\cap}(Z, E)]\tilde{\cap}F_{1E} = bScl^s_\mu F_{2E}\tilde{\cap}F_{1E} = \tilde{\varphi} \Leftrightarrow F_{1E}, F_{2E}$ are supra soft b-separated on μ. □

Theorem 3.5. *Let (Z, E) be a soft subset of a supra soft topological space (X, μ, E). Then, Z_E is supra soft b-connected w.r.t (X, μ, E) if and only if it is supra soft b-connected w.r.t (Z, μ_Z, E).*

Proof. Suppose that Z_E is supra soft b-disconnected w.r.t $(Z, \mu_Z, E) \Leftrightarrow Z_E = F_{1E}\tilde{\cup}F_{2E}$, where F_{1E} and F_{2E} are supra soft b-separated on $\mu_Z \Leftrightarrow Z_E = F_{1E}\tilde{\cup}F_{2E}$, where F_{1E} and F_{2E} are supra soft b-separated on μ_Z from Theorem 3.4 $\Leftrightarrow Z_E$ is supra soft b-disconnected w.r.t (X, μ, E). □

Theorem 3.6. *Let (Z, μ_Z, E) be a supra soft b-connected subspace of a supra soft topological space (X, μ, E) and $F_E,\ G_E$ are supra soft b-separated sets of $\tilde{X}$ with $Z_E\tilde{\subseteq}F_E\tilde{\cup}G_E$, then either $Z_E\tilde{\subseteq}F_E$ or $Z_E\tilde{\subseteq}G_E$.*

Proof. Let $Z_E\tilde{\subseteq}F_E\tilde{\cup}G_E$ for some supra soft b-separated sets $F_E,\ G_E$ on μ. Since $Z_E = (Z_E\tilde{\cap}F_E)\tilde{\cup}(Z_E\tilde{\cap}G_E)$. Then, $(Z_E\tilde{\cap}F_E)\tilde{\cup}bScl^s_\mu(Z_E\tilde{\cap}G_E)\tilde{\subseteq}F_E\tilde{\cap}bScl^s_\mu(G_E) = \tilde{\varphi}$. Also, $bScl^s_\mu(Z_E \tilde{\cap}F_E)\tilde{\cup}(Z_E\tilde{\cap}G_E)\tilde{\subseteq}cl_\mu(F_E)\tilde{\cap}G_E = \tilde{\varphi}$. If $Z_E\tilde{\cap}F_E$ and $Z_E\tilde{\cap}G_E$ are non-null soft sets. Then, Z_E is supra soft b-disconnected, which is a contradiction with the hypothesis. Thus, either $Z_E\tilde{\cap}F_E = \tilde{\varphi}$ or $Z_E\tilde{\cap}G_E = \tilde{\varphi}$. It follows that, $Z_E = Z_E\tilde{\cap}F_E$ or $Z_E = Z_E\tilde{\cap}G_E$. Therefore, $Z_E\tilde{\subseteq}F_E$ or $Z_E\tilde{\subseteq}G_E$. □

Corollary 3.3. The supra soft b-closure of a supra soft b-connected set is supra soft b-connected.

Proof. It is follows from Theorem 3.6. □

Theorem 3.7.

- **(1):** *Every supra soft b-component of a supra soft topological space (X, μ, E) is a maximal supra soft b-connected subset of $\tilde{X}$.*
- **(2):** *Every supra soft b-component of a supra soft topological space (X, μ, E) is a supra b-closed soft set.*

Proof. It is obvious from Definition 3.8 and Corollary 3.3. □

Theorem 3.8. *Let F_E be supra soft b-connected subsets of a supra soft topological space (X, μ, E) and G_E be a soft set such that $F_E\tilde{\subseteq}G_E\tilde{\subseteq}bScl^s(F_E)$, then G_E is supra soft b-connected.*

Proof. Let G_E be a supra soft b-disconnected, then there exist two non-null supra b-open soft sets U_E and V_E such that $G_E = U_E\tilde{\cup}V_E$. Since $F_E\tilde{\subseteq}G_E$ and F_E be supra soft b-connected, then by using Lemma 4.10 either $F_E\tilde{\subseteq}U_E$ or $F_E\tilde{\subseteq}V_E$. If $F_E\tilde{\subseteq}U_E$, then $bScl^s(F_E)\tilde{\subseteq}bScl^s(U_E)$ and so $bScl^s(F_E)\tilde{\cap}V_E = \tilde{\varphi}$ i.e, $V_E\tilde{\subseteq}\tilde{X_E} - bScl^sF_E)$, but $V_E\tilde{\subseteq}G_E \tilde{\subseteq}bScl^s(F_E)$. Thus, $V_E = \tilde{\varphi}$, which is a contradiction, and so G_E is supra soft b-connected. Similarly, if $F_E\tilde{\subseteq}V_E$, thus $U_E = \tilde{\varphi}$ this is a contradiction. Consequently, G_E is supra soft b-connected. □

Theorem 3.9. *Let (Z, μ_Z, E) be a supra soft b-connected subspace of a supra soft b-connected topological space (X, μ, E) such that Z^c_E is the soft union of two supra soft b-separated sets $F_E,\ G_E$ of $\tilde{X}$, then $Z_E\tilde{\cup}F_E$ and $Z_E\tilde{\cup}G_E$ are supra soft b-connected.*

Proof. The reader can prove it by using Theorem 3.6 and [Theorem 4.2 (1), [1]]. □

Theorem 3.10. *If Z_E, Y_E are supra soft b-connected sets such that none of them is supra soft b-separated sets, then $G_E\tilde{\cup}H_E$ is supra soft b-connected set.*

Proof. Immediate from Theorem 3.6. □

Theorem 3.11. *If for all pair of soft points $x_\alpha,\ y_\beta\tilde{\in}X$ with $x_\alpha \neq y_\beta$ there exists a supra soft b-connected set $(Z, E) \subseteq \tilde{X}$ with $x_\alpha,\ y_\beta\tilde{\in}(Z, E)$, then $\tilde{X}$ is supra soft b-connected.*

Proof. Suppose that $\tilde{X}$ is supra soft b-disconnected. Then, $\tilde{X} = (F,E)\tilde{\cup}(G,E)$, for some $(F,E),(G,E)$ supra soft b-separated sets. It follows that, $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi}$. So, $\exists\ x_\alpha\tilde{\in}(F,E)$ and $y_\beta\tilde{\in}(G,E)$. Since $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi}$. Then, $x_\alpha,\ y_\beta\tilde{\in}X$ with $x_\alpha \neq y_\beta$. By hypothesis, there exists a supra soft b-connected set $(Z,E) \subseteq \tilde{X}$ with $x_\alpha,\ y_\beta\tilde{\in}(Z,E)$. Moreover, we have (Z,E) is supra soft b-connected subset of a supra soft b-disconnected space. By Theorem 3.6, either $(Z,E)\tilde{\subseteq}(F,E)$ or $(Z,E)\tilde{\subseteq}(G,E)$, and both cases is a contradiction with the hypothesis. This implies that, $\tilde{X}$ is supra soft b-connected. □

Definition 3.9. A soft set N_E is said to be a supra soft b-neighborhood (briefly, supra soft b-nbd.) of a soft point $x_e\tilde{\in}(X,E)$ if there exists a supra b-open soft set $U_E\tilde{\subseteq}N_E$ such that $x_e\tilde{\in}U_E\tilde{\subseteq}N_E$.

Definition 3.10. A soft point $x_e\tilde{\in}(X,E)$is called supra soft b-limit point of a soft set F_E if every supra soft b-nbd U_E of x_e contains a point of F_E other than x_e.

Theorem 3.12. *Let F_E and G_E be non-null disjoint soft sets of a supra soft topological space (X,μ,E) and $Y_E = F_E\tilde{\cup}G_E$. Then, F_E and G_E are supra soft b-separated if and only if each of F_E and G_E is supra b-closed soft (supra b-open soft) with respect to Y_E.*

Proof. Follows from Definition 2.4 and Theorem 3.11. □

Theorem 3.13. *The soft union of any family of supra soft b-connected sets having a non-null soft intersection is supra soft b-connected set.*

Proof. Follows from Theorem 3.6. □

Proposition 3.1. *Let $\{(Z_j,\mu_{Z_j},E)$ be a family of supra soft b-connected subspaces of supra soft topological space (X,μ,E) such that one of the members of the family intersects every other members, then $(\tilde{\cup}_{j\in J}Z_j,\mu_{\tilde{\cup}_{j\in J}Z_j},E)$ is supra soft b-connected.*

Proof. Let $(Z,\mu_Z,E) = (\tilde{\cup}_{j\in J}Z_j,\mu_{\tilde{\cup}_{j\in J}Z_j},E)$ and $(Z_{jo},E) \in \{(Z_j,E) : j \in J\}$ such that $(Z_{jo},E)\tilde{\cap}(Z_j,E) \neq \tilde{\varphi}\ \ \forall j \in J$. Then, $(Z_{jo},E)\tilde{\cup}(Z_j,E)$ is supra soft b-connected $\forall j \in J$ from Theorem 3.13. Therefore, the collection $\{(Z_{jo},E)\tilde{\cup}(Z_j,E) : j \in J\}$ is a collection of a supra soft b-connected subsets of $\tilde{X}$, which having a non-null soft intersection. Thus,$(\tilde{\cup}_{j\in J}Z_j,\mu_{\tilde{\cup}_{j\in J}Z_j},E)$ is supra soft b-connected from Theorem 3.13. □

Definition 3.11. A supra soft topological space (X,μ,E) is said to be a supra soft locally b-connected at a soft point x_α if every supra soft b-nbd of the soft point x_α contains a supra soft b-connected nbd of x_α. $\tilde{X}$ is said to be a supra soft locally b-connected if it is supra soft locally b-connected at each of its soft points.

Proposition 3.2. *Every supra soft b-connected space is a supra soft locally b-connected space, but the converse is not true in general.*

Proof. Suppose that (X,μ,E) be a supra soft b-connected. Then, There is no proper supra b-clopen soft set in (X,μ,E) from [Theorem 5.1, [1]]. Hence, $\forall\ x_\alpha\tilde{\in}\tilde{X}\ \exists\ \tilde{X} \in \mu$ which is supra soft b-connected set such that $x_\alpha \in \tilde{X}\tilde{\subseteq}\tilde{X}$. Therefore, $\tilde{X}$ is supra soft locally b-connected. On the other hand, the indiscrete soft topological space, is supra soft locally b-connected but not supra soft b-connected. □

Theorem 3.14. *The supra soft b-component of a supra soft locally b-connected soft topological space is supra b-open soft set.*

Proof. Let (X,μ,E) be a supra soft locally b-connected , $x_\alpha \in \tilde{X}$ and $\tilde{SC}^s_b$ be a supra soft b-component of $\tilde{X}$ w.r.t x_α. Since (X,μ,E) is a supra soft locally b-connected space.

Therefore, every supra b-open soft set containing x_α contains a supra soft b-connected open set G_E containing x_α. But, $\tilde{SC}_b^s$ is the largest supra soft b-connected set containing x_α. Hence, $x_\alpha \in G_E \tilde{\subseteq} \tilde{SC}_b^s$, i.e $\tilde{SC}_b^s$ is a supra soft b-nbd of x_α. Thus, $\tilde{SC}_b^s$ is a supra soft b-nbd of each of its points. This means that, $\tilde{SC}_b^s$ is a supra b-open soft set. □

Theorem 3.15. *The property of supra soft locally b-connectedness is hereditary w.r.t supra b-open soft subspaces.*

Proof. Suppose that (Z, μ_Z, E) be a supra b-open soft subspace of a supra soft locally b-connected topological space (X, μ, E) and let $x_\alpha \in \tilde{Z}$. Since $\tilde{X}$ is supra soft locally b-connected. Then, $\exists\, G_E \in \mu$ such that G_E is μ-supra soft b-connected subset of $\tilde{X}$ and $x_\alpha \in G_E \tilde{\subseteq} \tilde{Z}$. Since $G_E \in \mu$ and G_E is μ-supra soft b-connected set. Then, $G_E \cap \tilde{\tilde{Z}} \in \mu_Z$ and $G_E \cap \tilde{Z}$ is μ_Y-supra soft b-connected subset of $\tilde{Z}$ by Theorem 3.5. So, $\tilde{Z}$ is supra soft locally b-connected for each $x_\alpha \in X$. Hence, $\tilde{Z}$ is supra soft locally b-connected. □

Theorem 3.16. *The supra soft b-components of every supra b-open soft subspace of a supra soft locally b-connected soft topological space are supra b-open soft.*

Proof. Immediate from Theorem 3.14 and Theorem 3.15. □

Definition 3.12. A supra soft topological space (X, μ, E) is said to be supra soft b-hyperconnected if and only if every pair of non-null proper supra b-open soft sets $(F, E), (G, E)$, has a non-null soft intersection, i.e (X, μ, E) is said to be supra soft b-hyperconnected if and only if for each $(F, E), (G, E) \in SBOS_E(X)$, we have $(F, E) \tilde{\cap} (G, E) \neq \tilde{\varphi}$.

Proposition 3.3. *Every supra soft b-hyperconnected soft topological space is supra soft b-connected.*

Proof. Clear. □

Proposition 3.4. *Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Then,*

(1) If $\tilde{X}$ is supra soft b-hyperconnected soft topological space, then it is soft hyperconnected.
(2) If $\tilde{X}$ is supra soft b-hyperconnected soft topological space, then it is soft connected.

Proof. Immediate. □

Corollary 3.4. *Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. The following implications hold from propositions 3.3, 3.4 and [Corollary 3.3,* [11]*].*

$\tilde{X}$ is supra soft b-hyperconnected $\Rightarrow$ $\tilde{X}$ is soft hyperconnected $\Downarrow$

$\Downarrow$

$\tilde{X}$ is supra soft b-connected $\Rightarrow$ $\tilde{X}$ is soft connected

Theorem 3.17. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a bijective supra b-irresolute soft function. If (G, A) is supra soft b-connected in X_1, then $f_{pu}(G, A)$ is supra soft b-connected in X_2.*

Proof. Suppose that $f_{pu}(G, A)$ is not supra soft b-connected in X_2. Then, $f_{pu}(G, A) = (M, B) \tilde{\cup} (N, B)$ for some supra soft b-separated sets $(M, B), (N, B)$ of $f_{pu}(G, A)$ in X_2 from [Theorem 5.1, [1]]. By [Theorem 4.4, [1]], $f_{pu}^{-1}(M, B)$ and $f_{pu}^{-1}(N, B)$ are supra soft b-separated in X. Since f_{pu} is bijective soft function. So, $(G, A) = f_{pu}^{-1}(f_{pu}(G, A)) = f_{pu}^{-1}(M, B) \tilde{\cup} f_{pu}^{-1}(N, B)$. It follows that, (G, A) is not supra soft b-connected in X_1, which is a contradiction. Thus, $f_{pu}(G, A)$ is supra soft b-connected in X_2. □

Corollary 3.5. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective supra b-irresolute soft function. If X_1 is supra soft b-connected space, then so X_2.*

Proof. Follows from Theorem 3.17. □

Theorem 3.18. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective supra b-continuous soft function. If (G, A) is supra soft b-connected in X_1, then $f_{pu}(G, A)$ is soft connected in X_2.*

Proof. The proof is similar to the proof of Theorem 3.17. □

Corollary 3.6. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective supra b-continuous soft function. If X_1 is supra soft b-connected space, then X_2 is soft connected.*

Proof. Follows from Theorem 3.18. □

Theorem 3.19. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a bijective b-closed soft function. If (H, B) is supra soft b-connected in X_2, then $f_{pu}^{-1}(H, B)$ is soft connected in X_1.*

Proof. The proof is similar to the proof of Theorem 3.18. □

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