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# Coefficient estimates for a class of bi-univalent functions associated with quasi-subordination

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ABSTRACT. In the present work, we define a new class associated with quasi-subordination and investigate the estimates on the first two coefficients $|a_2|$ and $|a_3|$. Some interesting applications of the results presented here are also discussed.

## 1. INTRODUCTION

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Further, by $\mathcal{S}$ we denote the family of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$. Let $h(z)$ be an analytic function in $\mathbb{U}$ and $|h(z)| \leq 1$, such that

$$h(z) = A_0 + A_1 z + A_2 z^2 + A_3 z^3 + \cdots , \tag{1.2}$$

where all coefficients are real. Also, let $\varphi$ be an analytic and univalent function with positive real part in $\mathbb{U}$ with $\varphi(0) = 1$, $\varphi'(0) > 0$ and $\varphi$ maps the unit disk $\mathbb{U}$ onto a region starlike with respect to $1$, and symmetric with respect to the real axis. The Taylor's series expansion of such function is of the form

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots , \tag{1.3}$$

where all coefficients are real and $B_1 > 0$. Throughout this paper we assume that the functions $h$ and $\varphi$ satisfy the above conditions one or otherwise stated.

For two functions $f$ and $g$, analytic in $\mathbb{U}$, we say that the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$, and write

$$f(z) \prec g(z) \qquad (z \in \mathbb{U})$$

if there exists a Schwarz function $w$, analytic in $\mathbb{U}$, with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1 \qquad (z \in \mathbb{U})$$

such that

$$f(z) = g(w(z)) \qquad (z \in \mathbb{U}) .$$

In particular, if the function $g$ is univalent in $\mathbb{U}$, the above subordination is equivalent to

$$f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

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For analytic functions $f$ and $g$, the function $f$ is quasi-subordinate to $g$ in the open unit disc $\mathbb{U}$, if there exist analytic functions $h$ and $w$, with $|h(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$, such that $\frac{f(z)}{h(z)}$ is analytic in $\mathbb{U}$ and written as

$$\frac{f(z)}{h(z)} \prec g(z) \qquad (z \in \mathbb{U}).$$

We also denote the above expression by

$$f(z) \prec_q g(z) \qquad (z \in \mathbb{U})$$

and this is equivalent to

$$f(z) = h(z)g(w(z)) \qquad (z \in \mathbb{U}).$$

Observe that if $h(z) \equiv 1$, then $f(z) = g(w(z))$, so that $f(z) \prec g(z)$ in $\mathbb{U}$. Also notice that if $w(z) = z$, then $f(z) = h(z)g(z)$ and it is said that $f$ is majorized by $g$ and written $f(z) \ll g(z)$ in $\mathbb{U}$. Hence it is obvious that quasi - subordination is a generalization of subordination as well as majorization (see [16]).

In [10] Ma and Minda, introduced the unified classes $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ given below:

$$\mathcal{S}^*(\varphi) := \left\{ f : f \in \mathcal{A} \quad \text{and} \quad \frac{zf'(z)}{f(z)} \prec \varphi(z); \quad z \in \mathbb{U} \right\} \tag{1.4}$$

and

$$\mathcal{K}(\varphi) := \left\{ f : f \in \mathcal{A} \text{ and } \quad 1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z); \quad z \in \mathbb{U} \right\}. \tag{1.5}$$

For the choice

$$\varphi(z) = \frac{1 + (1 - 2\alpha)z}{1 - z} \qquad (0 \leq \alpha < 1) \tag{1.6}$$

or

$$\varphi(z) = \left( \frac{1 + z}{1 - z} \right)^{\beta} \qquad (0 < \beta \leq 1) \tag{1.7}$$

the classes $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ consist of functions known as the starlike (respectively convex) functions of order $\alpha$ or strongly starlike (respectively convex) functions of order $\beta$ respectively. Further, analogous to Ma-Minda starlike and convex classes, Mohd and Darus [12] considered the notion of the quasi - subordination and introduced the classes $\mathcal{S}_q^*(\varphi)$ and $\mathcal{K}_q(\varphi)$ given below:

$$\mathcal{S}_q^*(\varphi) := \left\{ f : f \in \mathcal{A} \quad \text{and} \quad \frac{zf'(z)}{f(z)} - 1 \prec_q \varphi(z) - 1; \quad z \in \mathbb{U} \right\} \tag{1.8}$$

and

$$\mathcal{K}_q(\varphi) := \left\{ f : f \in \mathcal{A} \text{ and } \quad \frac{zf''(z)}{f'(z)} \prec_q \varphi(z) - 1; \quad z \in \mathbb{U} \right\}. \tag{1.9}$$

Following, Mohd and Darus [12], many researchers used the notion of the quasi - subordination to introduce several classes one could refer [6, 8, 11] and the references therein.

It is well known that every function $f \in \mathcal{S}$ has an inverse $f^{-1}$, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w \qquad \left( |w| < r_0(f); \quad r_0(f) \geq \frac{1}{4} \right),$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{1.10}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$, if both $f$ and $f^{-1}$ are univalent in $\mathbb{U}$. Let $\sigma$ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). For a brief history and interesting examples of functions which are in (or which are not in) the class $\sigma$, together with various other properties of the bi-univalent function class $\sigma$ one can refer the work of Srivastava et al. [18] and references therein. Recently, various subclasses of the bi-univalent function class $\sigma$ were introduced and non-sharp estimates on the first two coefficients $|a_2|$ and $|a_3|$ in the Taylor–Maclaurin series expansion (1.1) were found in several recent investigations (see, for example, [1, 2, 3, 4, 11, 13, 17, 19]). However, not much was known about the bounds of the general coefficients $a_n; n \geq 4$ for functions $f \in \sigma$ up until the work by Jahangiri and Hamidi [9]. They obtained bounds for the $n-$th coefficients $a_n; n \geq 3$ of certain subclasses of bi-univalent functions using the Faber polynomial series expansions subject to a given gap series condition. But, the problem to find the coefficient bounds on $|a_n|$ $(n = 3, 4, \dots)$ for functions $f \in \sigma$ is still an open problem.

In this paper we define the following subclass of the function class $\sigma$:

A function $f \in \sigma$ given by (1.1) is said to be in the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi)$ if the following quasi - subordination conditions are satisfied:

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} - 1 \prec_q \varphi(z) - 1 \qquad (\lambda \geq 1,\ \mu \geq 0,\ z \in \mathbb{U}) \quad (1.11)$$

and

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} - 1 \prec_q \varphi(w) - 1 \qquad (\lambda \geq 1,\ \mu \geq 0,\ w \in \mathbb{U}), \quad (1.12)$$

where $g = f^{-1}$.

**Remark 1.1.** From among the many choices of $\mu$, $\lambda$ and the function $\varphi$ which would provide the following new and known subclasses:

(1) $\mathcal{N}^{1,\lambda}_{q,\sigma}(\varphi) = \mathcal{R}_{q,\sigma}(\lambda, \varphi)$ $(\lambda \geq 0)$ [5]
(2) $\mathcal{N}^{\mu,1}_{q,\sigma}(\varphi) = \mathcal{F}^{\mu}_{q,\sigma}(\varphi)$ $(\mu \geq 0)$ [7]
(3) $\mathcal{N}^{0,1}_{q,\sigma}(\varphi) = \mathcal{S}^{*}_{q,\sigma}(\varphi)$
(4) $\mathcal{N}^{1,1}_{q,\sigma}(\varphi) = \mathcal{H}^{\varphi}_{q,\sigma}$.

For $h(z) \equiv 1$ the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi) := \mathcal{N}^{\mu,\lambda}_{\sigma}(\varphi)$ was considered by Tang et al. [19] and Orhan et al. [13, 14] to obtain bounds on initial coefficients $|a_2|$ and $|a_3|$. Further, for $\varphi$ given by (1.6) in the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi)$ with $h(z) \equiv 1$ and for $\varphi$ given by (1.7) in the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi)$ with $h(z) \equiv 1$ were considered by Çağlar et al. [2]. Also, the class was generalized by Srivastava et al. [17]. Motivated in this line we define the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi)$ and obtain the estimates on initial coefficients of normalized analytic function $f$ in the open unit disk with $f$ and its inverse $g = f^{-1}$ satisfying the conditions given in (1.11) and (1.12) are both quasi-subordinate to a univalent function whose range is symmetric with respect to the real axis. In order to derive our results, we need the following lemma.

**Lemma 1.1.** (see [15]) *If $p \in \mathcal{P}$, then $|p_i| \leq 2$ for each $i$, where $\mathcal{P}$ is the family of all functions $p$, analytic in $\mathbb{U}$, for which*

$$\Re\{p(z)\} > 0 \quad (z \in \mathbb{U}),$$

*where*

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots \quad (z \in \mathbb{U}).$$

## 2. Initial coefficient estimates for the class $\mathcal{N}_{q,\sigma}^{\mu,\lambda}(\varphi)$

**Theorem 2.1.** *Let $f$ of the form* (1.1) *be in $\mathcal{N}_{q,\sigma}^{\mu,\lambda}(\varphi)$. Then*

$$|a_2| \leq \frac{|A_0|B_1\sqrt{2B_1}}{\sqrt{|A_0B_1^2(2\lambda+\mu)(1+\mu)-2(B_2-B_1)(\lambda+\mu)^2|}} \tag{2.13}$$

*and*

$$|a_3| \leq \begin{cases} \frac{|A_1|B_1}{2\lambda+\mu} + \frac{2|A_0|[B_1+|B_2-B_1|]}{(2\lambda+\mu)(1+\mu)}, & 0 \leq \mu < 1 \\ \frac{|A_1|B_1}{2\lambda+\mu} + \frac{|A_0|B_1}{2\lambda+\mu} + \frac{2|A_0||B_2-B_1|}{(2\lambda+\mu)(1+\mu)}, & \mu \geq 1. \end{cases} \tag{2.14}$$

*Proof.* Since $f \in \mathcal{N}_{q,\sigma}^{\mu,\lambda}(\varphi)$, there exists two analytic functions $r, s : \mathbb{U} \to \mathbb{U}$, with $r(0) = 0$ and $s(0) = 0$, such that

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} - 1 = h(z)(\varphi(r(z))-1) \tag{2.15}$$

and

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} - 1 = h(w)(\varphi(s(w))-1). \tag{2.16}$$

Define the functions $u$ and $v$ by

$$u(z) = \frac{1+r(z)}{1-r(z)} = 1 + u_1z + u_2z^2 + u_3z^3 + \cdots \tag{2.17}$$

and

$$v(z) = \frac{1+s(z)}{1-s(z)} = 1 + v_1z + v_2z^2 + v_3z^3 + \cdots \tag{2.18}$$

or equivalently,

$$\begin{aligned} r(z) &= \frac{u(z)-1}{u(z)+1} = \frac{1}{2}\left(u_1z + \left(u_2 - \frac{u_1^2}{2}\right)z^2\right. \\ &\quad \left. + \left(u_3 + \frac{u_1}{2}\left(\frac{u_1^2}{2} - u_2\right) - \frac{u_1u_2}{2}\right)z^3 + \cdots\right) \end{aligned} \tag{2.19}$$

and

$$\begin{aligned} s(z) &= \frac{v(z)-1}{v(z)+1} = \frac{1}{2}\left(v_1z + \left(v_2 - \frac{v_1^2}{2}\right)z^2\right. \\ &\quad \left. + \left(v_3 + \frac{v_1}{2}\left(\frac{v_1^2}{2} - v_2\right) - \frac{v_1v_2}{2}\right)z^3 + \cdots\right). \end{aligned} \tag{2.20}$$

Using (2.19) and (2.20) in ( 2.15) and (2.16), we have

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} - 1 = h(z)\left[\varphi\left(\frac{u(z)-1}{u(z)+1}\right) - 1\right] \tag{2.21}$$

and

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} - 1 = h(w)\left[\varphi\left(\frac{q(w)-1}{q(w)+1}\right) - 1\right]. \tag{2.22}$$

Again using (2.19) and (2.20) along with (1.3), it is evident that

$$\begin{aligned} & h(z)\left[\varphi\left(\frac{u(z)-1}{u(z)+1}\right) - 1\right] \\ &= \frac{1}{2}A_0B_1u_1z + \left(\frac{1}{2}A_1B_1u_1 + \frac{1}{2}A_0B_1\left(u_2 - \frac{1}{2}u_1^2\right) + \frac{1}{4}A_0B_2u_1^2\right)z^2 + \cdots \end{aligned} \tag{2.23}$$

and

$$h(w)\left[\varphi\left(\frac{q(w)-1}{q(w)+1}\right)-1\right] \tag{2.24}$$
$$=\frac{1}{2}A_0B_1v_1w+\left(\frac{1}{2}A_1B_1v_1+\frac{1}{2}A_0B_1\left(v_2-\frac{1}{2}v_1^2\right)+\frac{1}{4}A_0B_2v_1^2\right)w^2+\cdots.$$

It follows from (2.21), (2.22), (2.23) and (2.24) that

$$(\lambda+\mu)a_2=\frac{1}{2}A_0B_1u_1 \tag{2.25}$$

$$(2\lambda+\mu)[a_3+\frac{a_2^2}{2}(\mu-1)]=\frac{1}{2}A_1B_1u_1+\frac{1}{2}A_0B_1\left(u_2-\frac{1}{2}u_1^2\right)+\frac{1}{4}A_0B_2u_1^2 \tag{2.26}$$

$$-(\lambda+\mu)a_2=\frac{1}{2}A_0B_1v_1 \tag{2.27}$$

and

$$(2\lambda+\mu)[\frac{a_2^2}{2}(\mu+3)-a_3]=\frac{1}{2}A_1B_1v_1+\frac{1}{2}A_0B_1\left(v_2-\frac{1}{2}v_1^2\right)+\frac{1}{4}A_0B_2v_1^2. \tag{2.28}$$

From (2.25) and (2.27), we find that

$$a_2=\frac{A_0B_1u_1}{2(\lambda+\mu)}=\frac{-A_0B_1v_1}{2(\lambda+\mu)} \tag{2.29}$$

it follows that

$$u_1=-v_1 \tag{2.30}$$

and

$$8(\lambda+\mu)^2a_2^2=A_0^2B_1^2(u_1^2+v_1^2). \tag{2.31}$$

Adding (2.26) and (2.28), we have

$$a_2^2(2\lambda+\mu)(\mu+1)=\frac{A_0B_1}{2}(u_2+v_2)+\frac{A_0(B_2-B_1)}{4}(u_1^2+v_1^2). \tag{2.32}$$

Substituting (2.29) and (2.30) into (2.32), we get,

$$u_1^2=\frac{2B_1(\lambda+\mu)^2(u_2+v_2)}{A_0B_1^2(2\lambda+\mu)(\mu+1)-2(B_2-B_1)(\lambda+\mu)^2}. \tag{2.33}$$

Now (2.29) and (2.33) yield

$$a_2^2=\frac{A_0^2B_1^3(u_2+v_2)}{2[A_0B_1^2(2\lambda+\mu)(\mu+1)-2(B_2-B_1)(\lambda+\mu)^2]}. \tag{2.34}$$

Applying Lemma 1.1 in (2.34), we get desired inequality (2.13). By subtracting (2.26) from (2.28) and a computation using (2.30) finally lead to

$$a_3=a_2^2+\frac{A_1B_1u_1}{2(2\lambda+\mu)}+\frac{A_0B_1(u_2-v_2)}{8\lambda+4\mu}. \tag{2.35}$$

Again applying Lemma 1.1, the equation (2.35) yields desired inequality (2.14). This completes the proof of Theorem 2.1. □

**Corollary 2.1.** *If $f\in\mathcal{S}^*_{q,\sigma}(\varphi)$, then*

$$|a_2|\leq\frac{|A_0|B_1\sqrt{B_1}}{\sqrt{|A_0B_1^2-B_2+B_1|}}$$

*and*

$$|a_3|\leq\frac{|A_1|B_1}{2}+|A_0|[B_1+2|B_2-B_1|].$$

**Remark 2.2.** Corollary 2.1 reduces to [7, Corollary 2.3, p.82].

**Corollary 2.2.** *If $f \in \mathcal{R}_{q,\sigma}(\lambda, \varphi)$, then*

$$|a_2| \leq \frac{|A_0| B_1 \sqrt{B_1}}{\sqrt{|A_0 B_1^2 (2\lambda + 1) - (B_2 - B_1)(\lambda + 1)^2|}}$$

*and*

$$|a_3| \leq \frac{|A_1| B_1 + |A_0| [B_1 + |B_2 - B_1|]}{2\lambda + 1}.$$

**Corollary 2.3.** *If $f \in \mathcal{F}^{\mu}_{q,\sigma}(\varphi)$, then*

$$|a_2| \leq \frac{|A_0| B_1 \sqrt{2B_1}}{\sqrt{|A_0 B_1^2 (2 + \mu)(1 + \mu) - 2(B_2 - B_1)(1 + \mu)^2|}}$$

*and*

$$|a_3| \leq \begin{cases} \frac{|A_1| B_1}{2+\mu} + \frac{2|A_0|[B_1 + |B_2 - B_1|]}{(2+\mu)(1+\mu)}, & 0 \leq \mu < 1 \\ \frac{|A_1| B_1}{2+\mu} + \frac{|A_0| B_1}{2+\mu} + \frac{2|A_0||B_2 - B_1|}{(2+\mu)(1+\mu)}, & \mu \geq 1. \end{cases}$$

**Remark 2.3.** The inequalities discussed in Corollary 2.3 improve the results obtained in [7, Theorem 2.1, p.80].

**Corollary 2.4.** *If $f \in \mathcal{H}_{q,\sigma}(\varphi)$, then*

$$|a_2| \leq \frac{|A_0| B_1 \sqrt{B_1}}{\sqrt{|3A_0 B_1^2 - 4(B_2 - B_1)|}}$$

*and*

$$|a_3| \leq \frac{1}{3} [|A_1| B_1 + |A_0| (B_1 + |B_2 - B_1|)].$$

**Remark 2.4.** The estimate $|a_2|$ obtained in Corollary 2.4 coincides with the estimate of [7, Corollary 2.6, p.84].

**Remark 2.5.** For $f$ given by (1.1) in the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi)$ with $h(z) \equiv 1$, the inequalities (2.13) and (2.14) reduce to the result in [19]. Further, for $\varphi$ given by (1.6) in the class $\mathcal{N}^{\mu,\lambda}_{q,\sigma}(\varphi)$ with $h(z) \equiv 1$, the inequalities (2.13) and (2.14) reduce to the result in [2] and for $h(z) \equiv 1$, and $\varphi$ given by (1.7) the inequalities (2.13) and (2.14) reduce to the result in [2].

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