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Fractional integral inequalities for $\mathbb{B}$-convex functions

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ABSTRACT. In this article, using Riemann-Liouville fractional integral, we obtain new generalized type inequalities of Hermite-Hadamard for $\mathbb{B}-$convex functions.

1. INTRODUCTION

Convexity and abstract convexity has very important place in mathematics and they have significant applications like mathematical economy, optimization, operation research. Especially, applications in the inequality theory always have a popularity and value in all the scientific background. If new developments in integral theory are taken into consideration, it can be seeing that examining fractional integral inequalities is normal. Therefore, we start to investigate the fractional integral inequalities for $\mathbb{B}-$convex functions.

$\mathbb{B}-$convexity is an abstract convexity type. In [7], W. Briec and C. D. Horvath introduced $\mathbb{B}-$convexity and its structure. Then various working about $\mathbb{B}-$convex sets, $\mathbb{B}$-convex functions, separation in $\mathbb{B}-$convexity, halfspaces and Hahn-Banach like properties in $\mathbb{B}-$convexity were published ([4, 6, 7, 8, 11, 18]). Its applications to the mathematical economy via data envelopment analysis were given in [9, 10].

Hermite-Hadamard Inequality is one of the important applications of convex functions to the Inequality Theory. For convex functions, the Hermite-Hadamard Inequality was proved in [15, 16]. Besides its improvements, also its versions for various abstract convex functions were studied([1, 2, 3, 26, 5, 14, 21, 25]). Hermite-Hadamard Inequalities for $\mathbb{B}-$convex functions were analysed in [26].

Hermite-Hadamard Inequality is required for the calculation of integral mean value that is used a good number of fields. But recently, we have needed more general inequalities involving new integral types that is fractional integrals. Thus, for the first time, fractional Hermite-Hadamard Inequalities were examined in [24]. Afterwards, Hermite-Hadamard type inequalities via fractional integrals for different abstract convex functions were studied ([12, 13, 20, 27]). For example, the fractional integral inequality was given for prequasiinvex functions in [17], for twice differentiable s-convex functions in [22], for MT-convex functions in [19].

Therefore, in this paper, we obtain fractional integral inequalities of Hermite-Hadamard type for $\mathbb{B}-$convex functions that are generalizations of the inequality proven in ([26]).

2. PRELIMINARIES

2.1. **Riemann-Liouville Fractional Integral.** When we express the fractional integral inequalities of Hermite-Hadamard type for $\mathbb{B}-$convex functions, we use the Riemann-Liouville fractional integral which is defined as follows.

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Definition 2.1. [23] Let $f : [a,b] \to \mathbb{R}$ be a given function, where $0 < a < b < \infty$ and $f \in L_1[a,b]$. The Riemann-Liouville integrals $J^{\alpha}_{a+}f$ and $J^{\alpha}_{b-}f$ of order $\alpha > 0$ are defined by

$$J^{\alpha}_{a+}f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)\,dt, \quad x > a$$

and

$$J^{\alpha}_{b-}f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t-x)^{\alpha-1} f(t)\,dt, \quad x < b$$

respectively. Here, $\Gamma(\alpha)$ is the Gamma function and $J^0_{a+}f(x) = J^0_{b-}f(x) = f(x)$.

2.2. **$\mathbb{B}$-convexity.** Let $r \in \mathbb{N}$, $\varphi_r : \mathbb{R} \to \mathbb{R}$, $\varphi_r(x) = x^{2r+1}$ and $\Phi_r : \mathbb{R}^n \to \mathbb{R}^n$, $\Phi_r(\boldsymbol{x}) = \Phi_r(x_1, x_2, ..., x_n) = (\varphi_r(x_1), \varphi_r(x_2), ..., \varphi_r(x_n))$. For a finite nonempty set $A = \{\boldsymbol{x^{(1)}}, \boldsymbol{x^{(2)}}, \cdots, \boldsymbol{x^{(m)}}\} \subset \mathbb{R}^n$, the r-convex hull of A, denoted as $Co^r(A)$, is given by

$$Co^r(A) = \left\{ \Phi_r^{-1}\left(\sum_{i=1}^m t_i \Phi_r(\boldsymbol{x^{(i)}}) \right) : t_i \geq 0, \sum_{i=1}^m t_i = 1 \right\}.$$

Definition 2.2. [7] The Kuratowski-Painleve upper limit of the sequence of sets $(Co^r(A))_{r\in\mathbb{N}}$, denoted by $Co^{\infty}(A)$ where A is a finite subset of $\mathbb{R}^n$, is called $\mathbb{B}$-polytope of A.

Definition 2.3. [7] A subset U of $\mathbb{R}^n$ is $\mathbb{B}$-convex if for all finite subset $A \subset U$ the $\mathbb{B}$-polytope $Co^{\infty}(A)$ is contained in U.

In $\mathbb{R}^n_+ = \{(x_1, ..., x_n) \in \mathbb{R}^n : x_i \geq 0,\ i = 1, 2, ..., n\}$, $\mathbb{B}$-convex set is defined in a different way [7]:

A subset U of $\mathbb{R}^n_+$ is $\mathbb{B}$-convex if and only if for all $\boldsymbol{x^{(1)}}, \boldsymbol{x^{(2)}} \in U$ and all $\lambda \in [0,1]$ one has $\lambda\boldsymbol{x^{(1)}} \vee \boldsymbol{x^{(2)}} \in U$.

Here, we denote the least upper bound with respect to the coordinate-wise order relation of $\boldsymbol{x^{(1)}}, \boldsymbol{x^{(2)}}, ..., \boldsymbol{x^{(m)}} \in \mathbb{R}^n$ by $\overset{m}{\underset{i=1}{\vee}} \boldsymbol{x^{(i)}}$, that is:

$$\overset{m}{\underset{i=1}{\vee}} \boldsymbol{x^{(i)}} = \left(\max\left\{x_1^{(1)}, x_1^{(2)}, ..., x_1^{(m)}\right\}, ..., \max\left\{x_n^{(1)}, x_n^{(2)}, ..., x_n^{(m)}\right\} \right)$$

where, $x_j^{(i)}$ denotes jth coordinate of the point $\boldsymbol{x^{(i)}}$.

Remark 2.1. In $\mathbb{R}_+$, $\mathbb{B}$-convex sets are intervals because of definition.

Furthermore, in [7, 18], the definition of $\mathbb{B}$-convex functions is given as follows:

Definition 2.4. Let $U \subset \mathbb{R}^n$. A function $f : U \to \mathbb{R}\bigcup\{\pm\infty\}$ is called a $\mathbb{B}$-convex function if $epi(f) = \{(\boldsymbol{x}, \mu) \,|\, \boldsymbol{x} \in U, \mu \in \mathbb{R}, \mu \geq f(\boldsymbol{x})\}$ is a $\mathbb{B}$-convex set.

The following theorem provides a sufficient and necessary condition for $\mathbb{B}$-convex functions in $\mathbb{R}^n_+$ [7, 18].

Theorem 2.1. *Let $U \subset \mathbb{R}^n_+$, $f : U \to \mathbb{R}_+ \cup \{+\infty\}$. The function f is $\mathbb{B}$-convex if and only if U is a $\mathbb{B}$-convex set and for all $\boldsymbol{x}, \boldsymbol{y} \in U$ and all $\lambda \in [0,1]$ the following inequality holds:*

$$f(\lambda\boldsymbol{x} \vee \boldsymbol{y}) \leq \lambda f(\boldsymbol{x}) \vee f(\boldsymbol{y}) \ . \tag{2.1}$$

2.3. **Hermite-Hadamard Inequality for $\mathbb{B}$-convex Functions.** Hermite-Hadamard Inequality for $\mathbb{B}$-convex functions that was given in the following form was examined.

Theorem 2.2. [26] *Let $f : [a,b] \subset \mathbb{R}_+ \to \mathbb{R}_+$ be a $\mathbb{B}$-convex function. Then one has the inequality*

$$\frac{1}{b-a}\int_a^b f(t)\,dt \leq \begin{cases} f(a), & 1 \leq \frac{f(a)}{f(b)} \\ \frac{b\left([f(a)]^2 + [f(b)]^2\right) - 2af(a)f(b)}{2(b-a)f(b)}, & 0 < \frac{f(a)}{f(b)} < 1 \ . \end{cases} \tag{2.2}$$

3. Fractional integral inequalities for $\mathbb{B}$-convex functions

Fractional integral inequalities type of Hermite-Hadamard for classes of abstract convex functions have been obtained. In this concept, it can also be examined for $\mathbb{B}$-convex functions. We can prove these new inequalities involving fractional integrals with following theorems.

Theorem 3.3. *Let $f : [a,b] \subset \mathbb{R}_+ \to \mathbb{R}_+$ and $f \in L_1[a,b]$. If f is a $\mathbb{B}$-convex function on $[a,b]$, then the following inequalities for fractional integrals hold:*

$$J_{a+}^{\alpha} f(b) \leq \begin{cases} \frac{f(a)(b-a)^{\alpha}}{\Gamma(\alpha+1)}, & 1 \leq \frac{f(a)}{f(b)} \\ \frac{b^{\alpha}(f(b)-f(a))^{\alpha+1}+(\alpha+1)f(a)(f(b))^{\alpha}(b-a)^{\alpha}}{\Gamma(\alpha+2)(f(b))^{\alpha}}, & 0 < \frac{f(a)}{f(b)} < 1 \end{cases} \tag{3.3}$$

with $\alpha > 0$.

Proof. Let $f : [a,b] \subset \mathbb{R}_+ \to \mathbb{R}_+$ be a $\mathbb{B}$-convex function. Then for all $\lambda \in [0,1]$ the following inequality holds:

$$f(\max\{a, \lambda b\}) \leq \max\{f(a), \lambda f(b)\} \; . \tag{3.4}$$

Multiplying both sides of (3.4) by $(1-\lambda)^{\alpha-1}$, and integrating the resulting inequality with respect to λ over $[0,1]$, we obtain the following for left hand side of the inequality:

$$\begin{aligned} \int_0^1 (1-\lambda)^{\alpha-1} f(\max\{a,\lambda b\})\, d\lambda &= \int_0^{\frac{a}{b}} (1-\lambda)^{\alpha-1} f(a)\, d\lambda + \int_{\frac{a}{b}}^1 (1-\lambda)^{\alpha-1} f(\lambda b)\, d\lambda \\ &= \frac{f(a)}{\alpha}\left(1 - \left(1-\frac{a}{b}\right)^{\alpha}\right) + \frac{\Gamma(\alpha)}{b^{\alpha}} J_{a+}^{\alpha} f(b). \end{aligned}$$

Also, for the right hand side of the inequality $\int_0^1 (1-\lambda)^{\alpha-1} \max\{f(a), \lambda f(b)\}\, d\lambda$, we have to be examine the following two cases.

1) $1 \leq \frac{f(a)}{f(b)}$. In this case, we get

$$\int_0^1 (1-\lambda)^{\alpha-1} \max\{f(a), \lambda f(b)\}\, d\lambda = \int_0^1 (1-\lambda)^{\alpha-1} f(a)\, d\lambda = \frac{f(a)}{\alpha}.$$

Then we deduce that

$$\int_0^1 (1-\lambda)^{\alpha-1} f(\max\{a,\lambda b\})\, d\lambda \leq \int_0^1 (1-\lambda)^{\alpha-1} \max\{f(a), \lambda f(b)\}\, d\lambda$$

$$\frac{f(a)}{\alpha}\left(1 - \left(1-\frac{a}{b}\right)^{\alpha}\right) + \frac{\Gamma(\alpha)}{b^{\alpha}} J_{a+}^{\alpha} f(b) \leq \frac{f(a)}{\alpha}$$

$$J_{a+}^{\alpha} f(b) \leq \frac{f(a)(b-a)^{\alpha}}{\Gamma(\alpha+1)}.$$

2) $0 < \frac{f(a)}{f(b)} < 1$, we have

$$\int_0^1 (1-\lambda)^{\alpha-1} \max\{f(a), \lambda f(b)\}\, d\lambda = \int_0^{\frac{f(a)}{f(b)}} (1-\lambda)^{\alpha-1} f(a)\, d\lambda +$$

$$\int_{\frac{f(a)}{f(b)}}^1 (1-\lambda)^{\alpha-1} \lambda f(b)\, d\lambda = \frac{(f(b)-f(a))^{\alpha+1}}{\alpha(\alpha+1)[f(b)]^{\alpha}} + \frac{f(a)}{\alpha}$$

Therefore, we obtain that

$$\int_0^1 (1-\lambda)^{\alpha-1} f(\max\{a,\lambda b\})\, d\lambda \leq \int_0^1 (1-\lambda)^{\alpha-1} \max\{f(a), \lambda f(b)\}\, d\lambda$$

$$\frac{f(a)}{\alpha}\left(1-\left(1-\frac{a}{b}\right)^{\alpha}\right)+\frac{\Gamma(\alpha)}{b^{\alpha}}J_{a+}^{\alpha}f(b)\leq\frac{(f(b)-f(a))^{\alpha+1}}{\alpha(\alpha+1)[f(b)]^{\alpha}}+\frac{f(a)}{\alpha}$$

$$J_{a+}^{\alpha}f(b)\leq\frac{b^{\alpha}(f(b)-f(a))^{\alpha+1}+(\alpha+1)f(a)(f(b))^{\alpha}(b-a)^{\alpha}}{\Gamma(\alpha+2)(f(b))^{\alpha}}.$$

□

Theorem 3.4. *Let $f:[a,b]\subset\mathbb{R}_+\to\mathbb{R}_+$ and $f\in L_1[a,b]$. If f is a $\mathbb{B}$-convex function on $[a,b]$, then the following inequalities for fractional integrals hold:*

$$J_{b-}^{\alpha}f(a)\leq\begin{cases}\frac{f(a)(b-a)^{\alpha}}{\Gamma(\alpha+1)}, & 1\leq\frac{f(a)}{f(b)}\\ \frac{(bf(a)-af(b))^{\alpha+1}+(f(b))^{\alpha+1}(b-a)^{\alpha}(\alpha b+a)}{b\Gamma(\alpha+2)(f(b))^{\alpha}}, & \frac{a}{b}\leq\frac{f(a)}{f(b)}<1\\ \frac{f(b)(b-a)^{\alpha}(\alpha b+a)}{b\Gamma(\alpha+2)}, & 0<\frac{f(a)}{f(b)}<\frac{a}{b}\end{cases}\tag{3.5}$$

with $\alpha>0$.

Proof. For the $\mathbb{B}$-convex function $f:[a,b]\subset\mathbb{R}_+\to\mathbb{R}_+$, we have the inequality (3.4), for all $\lambda\in[0,1]$. To prove the inequality (3.5), we have to multiply both sides of (3.4) by $\left(\lambda-\frac{a}{b}\right)^{\alpha-1}$. For $\lambda\in\left[0,\frac{a}{b}\right)$, since $\lambda-\frac{a}{b}<0$ all of the interval can't be take. Thus, we integrate the resulting inequality with respect to λ over $\left[\frac{a}{b},1\right]$:

$$\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}f(\max\{a,\lambda b\})\,d\lambda\leq\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}\max\{f(a),\lambda f(b)\}\,d\lambda\,.\tag{3.6}$$

We obtain the following for left hand side of inequality (3.6)

$$\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}f(\max\{a,\lambda b\})\,d\lambda=\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}f(\lambda b)\,d\lambda=\frac{\Gamma(\alpha)}{b^{\alpha}}J_{b-}^{\alpha}f(a)$$

Let us examine the rigth hand side of inequality (3.6). We should handle three cases:

1) $1\leq\frac{f(a)}{f(b)}$. Therefore, we obtain that

$$\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}\max\{f(a),\lambda f(b)\}\,d\lambda=\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}f(a)\,d\lambda=\frac{f(a)(b-a)^{\alpha}}{\alpha b^{\alpha}}\,.$$

Thus, the inequality (3.6) is

$$\frac{\Gamma(\alpha)}{b^{\alpha}}J_{b-}^{\alpha}f(a)\leq\frac{f(a)(b-a)^{\alpha}}{\alpha b^{\alpha}}J_{b-}^{\alpha}f(a)\leq\frac{f(a)(b-a)^{\alpha}}{\Gamma(\alpha+1)}.\tag{3.7}$$

2) In the second case, we can have $\frac{a}{b}\leq\frac{f(a)}{f(b)}<1$. Therefore, we deduce that

$$\int_{\frac{a}{b}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}\max\{f(a),\lambda f(b)\}\,d\lambda=\int_{\frac{a}{b}}^{\frac{f(a)}{f(b)}}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}\max\{f(a),\lambda f(b)\}\,d\lambda$$

$$+\int_{\frac{f(a)}{f(b)}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}\max\{f(a),\lambda f(b)\}\,d\lambda$$

$$=\int_{\frac{a}{b}}^{\frac{f(a)}{f(b)}}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}f(a)\,d\lambda+\int_{\frac{f(a)}{f(b)}}^{1}\left(\lambda-\frac{a}{b}\right)^{\alpha-1}\lambda f(b)\,d\lambda=\frac{f(a)}{\alpha}\left[\frac{f(a)}{f(b)}-\frac{a}{b}\right]^{\alpha}$$

$$+f(b)\left[\frac{\left(1-\frac{a}{b}\right)^{\alpha+1}}{\alpha+1}+\frac{a}{b}\frac{\left(1-\frac{a}{b}\right)^{\alpha}}{\alpha}-\frac{\left(\frac{f(a)}{f(b)}-\frac{a}{b}\right)^{\alpha+1}}{\alpha+1}-\frac{a}{b}\frac{\left(\frac{f(a)}{f(b)}-\frac{a}{b}\right)^{\alpha}}{\alpha}\right]$$

$$= \frac{(bf(a) - af(b))^{\alpha+1} + (f(b))^{\alpha+1} (b-a)^{\alpha} (\alpha b + a)}{\alpha(\alpha+1) b^{\alpha+1} (f(b))^{\alpha}}.$$

Thus, the inequality (3.6) is

$$\frac{\Gamma(\alpha)}{b^{\alpha}} J_{b-}^{\alpha} f(a) \leq \frac{(bf(a) - af(b))^{\alpha+1} + (f(b))^{\alpha+1} (b-a)^{\alpha} (\alpha b + a)}{\alpha(\alpha+1) b^{\alpha+1} (f(b))^{\alpha}}$$

$$J_{b-}^{\alpha} f(a) \leq \frac{(bf(a) - af(b))^{\alpha+1} + (f(b))^{\alpha+1} (b-a)^{\alpha} (\alpha b + a)}{b\Gamma(\alpha+2) (f(b))^{\alpha}}. \tag{3.8}$$

3) In case of $0 < \frac{f(a)}{f(b)} < \frac{a}{b}$, the right hand side of inequality (3.6) is

$$\begin{aligned} \int_{\frac{a}{b}}^{1} \left(\lambda - \frac{a}{b}\right)^{\alpha-1} \max\{f(a), \lambda f(b)\} d\lambda &= \int_{\frac{a}{b}}^{1} \left(\lambda - \frac{a}{b}\right)^{\alpha-1} \lambda f(b) d\lambda \\ &= f(b) \left[\frac{\left(1 - \frac{a}{b}\right)^{\alpha+1}}{\alpha+1} + \frac{a}{b} \frac{\left(1 - \frac{a}{b}\right)^{\alpha}}{\alpha} \right] \\ &= \frac{f(b)(b-a)^{\alpha} (\alpha b + a)}{\alpha(\alpha+1) b^{\alpha+1}}. \end{aligned}$$

Hence, we have that

$$\frac{\Gamma(\alpha)}{b^{\alpha}} J_{b-}^{\alpha} f(a) \leq \frac{f(b)(b-a)^{\alpha} (\alpha b + a)}{\alpha(\alpha+1) b^{\alpha+1}}$$

$$J_{b-}^{\alpha} f(a) \leq \frac{f(b)(b-a)^{\alpha} (\alpha b + a)}{b\Gamma(\alpha+2)}. \tag{3.9}$$

Finally, from (3.7), (3.8) and (3.9), we have that the Hermite-Hadamard Inequality with fractional form of $\mathbb{B}$-convex function is

$$J_{b-}^{\alpha} f(a) \leq \begin{cases} \frac{f(a)(b-a)^{\alpha}}{\Gamma(\alpha+1)}, & 1 \leq \frac{f(a)}{f(b)} \\ \frac{(bf(a)-af(b))^{\alpha+1} + (f(b))^{\alpha+1}(b-a)^{\alpha}(\alpha b + a)}{b\Gamma(\alpha+2)(f(b))^{\alpha}}, & \frac{a}{b} \leq \frac{f(a)}{f(b)} < 1 \\ \frac{f(b)(b-a)^{\alpha}(\alpha b + a)}{b\Gamma(\alpha+2)}, & 0 < \frac{f(a)}{f(b)} < \frac{a}{b} \end{cases}$$

□

Note that for $\alpha = 1$, the inequalities (3.3) and (3.5) reduces to the classic Hermite-Hadamard Inequality for $\mathbb{B}$-convex functions given with Theorem 2.2.

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