

Nicole Oresme's Quest towards the Realm of Reality: Are There Any Precursory Themes of Applied Mathematics Present in His Works?

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ABSTRACT. The present paradigm associates the dawn of modern applied mathematics with the first decades of the 19th century. In an investigation of these historical premises, we search for themes investigated today through methods pertaining to applied mathematics in the works of a medieval scholar whose singular vision helped him reach several conclusions that were definitely ahead of his time. Nicole Oresme's work, *Tractatus de configurationibus qualitatum et motuum*, written approximately between 1351 and 1355, showcases early mathematical applications that would now be classified as works in applied mathematics.

1. OUR MOTIVATION

Arguably, if works detailing the application of Archimedes' philosophy survived, the origin of applied mathematics could be traced to him. Yet, with few of his original works available, we are left asking: *when did philosophers begin to explain natural phenomena through mathematics?* Common perception holds that the Western European medieval sciences contributed little to the modern scientific revolution. Despite this notion, Edward Grant [5] asks: "Even if the Middle Ages made few significant contributions to the advancement of the sciences themselves, or none at all, [...] if no noteworthy medieval contributions were made to help shape specific scientific advances in the seventeenth century, in what ways did the Middle Ages contribute to the Scientific Revolution and, more to the point, lay the foundations for it?" [16]. Giving ode to the spirit of medieval inquiry, Richard Rubinstein [14] writes: "Two names are sufficient to explode the myth of medieval ignorance: Jean Buridan and Nicole Oresme," and further assesses that "like bold scientific thinkers ever since, they took an existing paradigm (the Aristotelian theory of motion) as their starting point, and ended by revolutionizing it." Our work focuses on philosopher Nicole Oresme, whose texts we survey to find evident parallels with contemporary applied mathematics. Contemplating Oresme's influence on the scientific revolution, Dirk Struik describes Oresme's work as "an exception", in the sense that it "forms a direct link with Renaissance science" [20]. As Oresme's work demonstrates, as early as the 14th century scientists made attempts to understand natural events through quantification and physical measurements. (See also [1, 6, 17].)

2. NICOLE ORESME'S BIOGRAPHY

Oresme was born around 1320 in the village of Allemagne, near Caen, or present-day Fleury-sur-Orne. Oresme was a "bursar" of the College of Navarre from 1348 to 1356, when he became a Master. Oresme's major was in Theology, but it is not known when he earned his degree of Master in Theology. As a student, he observed the College of

Received: 24.10.2021. In revised form: 24.01.2022. Accepted: 01.02.2022

2020 *Mathematics Subject Classification.* 01A35, 01A50, 92Bxx, 01A55, 01A65, 01A70.

Key words and phrases. *mathematical modeling, applied mathematics, Nicole Oresme, 14th Century.*

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Navarre's academic code, which required him to speak and write in Latin, and learn material by rote. At this institution he wrote his most important works, e.g. *De proportionibus proportionum*, which is of particular importance for the history of mathematics, and *Ad pauca respicientes* of interest for the history of ideas in celestial mechanics.

Oresme remained Master of the College until December 4, 1361, when, during complex political times, he was forced to resign. On November 23, 1362 he became a canon of the Rouen Cathedral, and on March 18, 1364 dean of the Cathedral. Researchers speculate that during this time Oresme served as the King's confessor and adviser, but his political influence is difficult to assess. Before 1370, Oresme acted as one of Charles V's (1364-1380) chaplains. At the king's request he translated, from Latin into French, Aristotle's *Ethics*, *Politics*, and *Economics*. As a result, he was considered an expert in Aristotle's work, at least in the circles at the French royal court.

In a related paper, the first two of the present authors discussed Nicole Oresme's anticipation of developments of curvature [16], which speaks plenty about Oresme's mathematical intuition. He was an extraordinary scholar, who anticipated many developments ahead of his time. In the present paper, we aim to add one more piece of evidence to his file.

3. *De configurationibus*

Marshall Clagett's critical edition of Oresme's *De configurationibus* [11] was published in 1968; Clagett's translations allows us to further investigate Oresme's work. *De configurationibus* has 93 chapters, and is composed of three parts. In the first part, Oresme sets the foundation for a "doctrine of configurations" and applies the doctrine to qualities, focusing on "entities" which are permanent or enduring in time. While discussing these elements, he suggests that his theory could explain numerous physical and psychological phenomena. In the second part, Oresme describes how graphical representation can be applied to "entities that are successive", particularly referring to motion. He concludes this part with examples illustrating the application of his theory to psychological effects, focusing on describing perceptions typically deemed as magical. Finally, in the third part, Oresme describes external geometrical figures used to represent qualities and motions. He explains that by comparing the areas of such figures one may have a basis for the comparison of different qualities and motions. Here, we have briefly summarized what he called "the doctrine of configurations," which can be viewed as an early theory of functions and of their graphical representations. Ultimately, Oresme's doctrine was formed on solid intuition as the rigorous algebraic notation and language of functions had not been developed. Oresme's insight into various topics in mathematics emphasizes the historical context of Oresme's work to highlight their potential applications.

4. NICOLE ORESME AND ARISTOTLE'S WORKS

Dirk Struik writes: "The study of the Aristotelian categories of quality and quantity focused attention on the difference between the *intensio* and *remissio* of a quality and the increase and decrease of a quantity," concluding that Oresme's pursuit "gave rise to the applications of quantitative ideas to qualities."

Oresme was inspired by Campanus (cited by Clagett [11]) who writes: "whatever ratio is found in one kind of continuum, the same ratio can be found in all others. For just as one line is related to any other line, so any surface is related to some other surface, and any body to some other body, and similarly for time." Oresme indicates that this was a highly influential source.

Additionally, Oresme pulled from Aristotle's *Metaphysics* (more precisely with Bk. X, Chap.I, 1052b), which is interpreted by Oresme in his *Questions on the Physics* in the following way:

... measure, ratio, comparison, equality, inequality, etc. are initially found in quantity and [then] transmuted to all other things by means of similarity to this quantity, either extended or discrete quantity. From this it is evident by corollary that comparison is initially in quantity and secondarily in [those] species of quantity like angles, and thirdly in qualities with respect to their intensity... Then finally I say by way of conclusion that in every comparison it is necessary to imagine extension, intensity, discreteness, or order, and I say that intensity is always imagined by means of extension. [11]

This connection with Aristotle's work clearly illustrates how Oresme builds upon Aristotle's conclusions, and applies these ideas to a larger array of concepts where proportions could be used.

5. ORESME'S APPLICATIONS

The generality of Oresme's doctrine resides in his efforts to model various phenomena. In *De configurationibus* we encounter his first attempt to apply his doctrine in chapter I.xxiv. He writes

It is manifest from natural philosophy and experience alike that all natural bodies determine in themselves their shapes, as, for example, animals, plants, some stones, and the parts of [all of] these. They also determine in themselves certain qualities which are natural to them. In addition to their shape that these qualities possess from their subject, it is necessary that they be figured with a figuration which they possess from their intensity - to employ the previously described imagery. [11]

Thus, the method of figurations developed before is meant to serve a series of representations associated to various phenomena. Only for this remark Oresme's work would be of major interest for the historian of mathematics, as such a description anticipates several developments of modern applied mathematics. However, there is more here, because each of these figurations have intensity, so additionally their shape needs to be understood. This idea is mathematical in nature and represents concepts ahead of Oresme's times.

For our inquiry, the key point resides in chapter I.xxxi, where Oresme writes "On difformity in cognitive powers." This is a matter that today could be associated either with psychology or with the physics of the brain. He writes,

Accidents of the sensitive soul are, in accordance with the extension of the subject, figured in the organs with respect to uniformity and difformity in completely the same way as are sensibles or the other qualities[...] [11].

And he continues:

forms impressed in the exterior senses pass away immediately when the sensibles are withdrawn. But in the interior sense likenesses or forms remain even in the absence of the sensibles, just as an imprint remains in wax after the seal has been removed. However, the organ of the interior sense is not in itself differently shaped according to quantity by the impressed species or form but is only figured qualitatively in the same way

that the corporeal figure of the eye is not in itself changed by receiving a species of color. [11]

What “accidents of the sensitive soul” does Oresme have in mind? His century offered him a wide array of personal experiences. The historical context of the major tragedies and experiences Oresme must have been subject to is partially an explanation of why this treatise is structured as such and why these problems are discussed in relation with mathematical concepts. We conclude that his work is a reflection of the 14th century, particularly corresponding with the beginning of the Hundred Years War and the Black Death.

However, the French court proved to complicate the creation of Oresme’s doctrine. C. W. Coopland writes:

As much of Oresme’s energy was directed to attach on “occult” practices and towards defining the bounds between what we should now distinguish as astronomy and astrology respectively, a word may be said as to the prevalence of superstition in his day. It was clearly widespread and mischievous. We need not stress too heavily evidence drawn from the titles of the books collected by Charles V in the great library installed by him in the Louvre. [2]

In part, Oresme was motivated to form his doctrine to contradict politicians that tried to control the king’s mind through superstitions and astrology. However, Oresme also sought to create a consistent “doctrine, which is mathematical in nature and substance.”

Oresme describes the matters of the “soul” as analogous to physical perceptions. In Oresme’s words,

consider leather in which writing or the image of some seal is impressed. [...] Because of the nature of the leather or the quality of the material and the strong impression, it happens that afterwards the leather can not be stretched without the impression always remaining visible. [...] Just as it is in the case of this quantitative figuration, so similarly we ought to imagine it to be in the case of qualitative figuration of sensitive power, and, in a certain corresponding way, in the case of spiritual figuration of intellective power. [11]

Thus, the main challenge set for the researcher of the “intellective power” is to understand the causes of “visions of the soul” (chapter I.xxxiii). In this chapter, the “things perceived by a soul through a vision, which is foreseen in dreams or in an ecstasy or excess of the mind,” are compared to images reflected into “a mirror.” However, mirrors could be “pure and clean” or “contaminated and infected.” Other mirrors are discussed in chapter I.xxxv, titled “On certain differences in visions,” as Oresme pursues his analysis: “certain mirrors are uniform and plane or straight, and others are curved. And the curved ones are of two kinds: either concave (and these magnify the object) or convex (and in these the object appears smaller in size).[...] It is the same also in regard to souls.” [11]

As shown in his analysis, Oresme needed the concepts of convexity, difformity, and curvature to investigate and explain perceptions. At stake: the ultimate truth. By developing a sound doctrine, Oresme had the potential to influence the king’s views and disprove dangerous political advisors. Oresme’s quest started as a question of intellectual nature – the relationship of senses with the real world. This question was reduced to an inquiry that today we would describe as a psychology problem addressed quantitatively. However, Oresme was writing for his 14th century audiences.

6. FRIENDSHIP

We are now ready to discuss another theme that our contemporary perspective finds of interest: the theme of friendship. This speaks plenty of Oresme's multifaceted contributions, which we intend to explore in our present paper, since the mathematical grounds of this theme is different from the one described previously. In Chapter I.xxvii, titled *On the beauty of figurations in relation to something else and on the causes of natural friendship and hostility*, Oresme writes that

one cause of the natural friendship between man and dog can be the fitting accord between the ratio of primary qualities in the human constitution and the ratio of the same qualities in that of the dog. Another cause can be the fitting accord between the configurations of the primary or other natural qualities in each of these species. I speak here of the fitting accord, not that of closeness, but that of conformity.

And, in the next paragraph, Oresme completes his argument by saying:

Similar discord (contrary to fitting accord) [between species] in respect to the ratios of qualities as well as to their figurations can be posited to be among the causes of natural hostility, as, for example, between a sheep and a wolf, or between some other animals differing in species.

Further in his work, Oresme writes in Chapter I.xxix, titled *On the causes of three kinds of friendship [between individuals] of the same species*, the following:

Three modes of natural friendship can be found [between individuals] of the same species of animal. The first is the general mode according to which one individual naturally likes another similar to it more than it does one dissimilar. The second mode is the special one existing between male and female, in which it appears that natural friendship is not always a function of the fitting accord of proximity and similarity but rather of that of conformity [of ratio and figuration], as has been said in chapter twenty-seven.

This discussion describes an interesting problem approached by contemporary research in the recent decades, namely the question of modeling affinity and friendship.

The study of friendship has its beginnings in the 1950s with the works of Fritz Heider [8, 9] on *structural balance*. Heider's theory on structural balance proposes that "sentiment" towards someone can be quantified by using a basic model of positive and negative relationships. As demonstrated in Figure 1, two people share an edge between them labeled with a + or - if they are friends or enemies, respectively. If the social group is formed of three people, say A, B, and C, there are four distinct ways, dependent on symmetry, to label the three edges among them. According to Heider, this social triangle will reach a balanced classification of sentiment between its members, which will be solely determined by the +'s and -'s along the edges. Heider's theory of structural balance resonates loudly with Oresme's writings in Chapter I.xxix above. As explained by Oresme "natural friendship is not always a function of the fitting accord of proximity," which in terms of Heider's theory means that sharing an edge does not inherently result in a positive relationship.

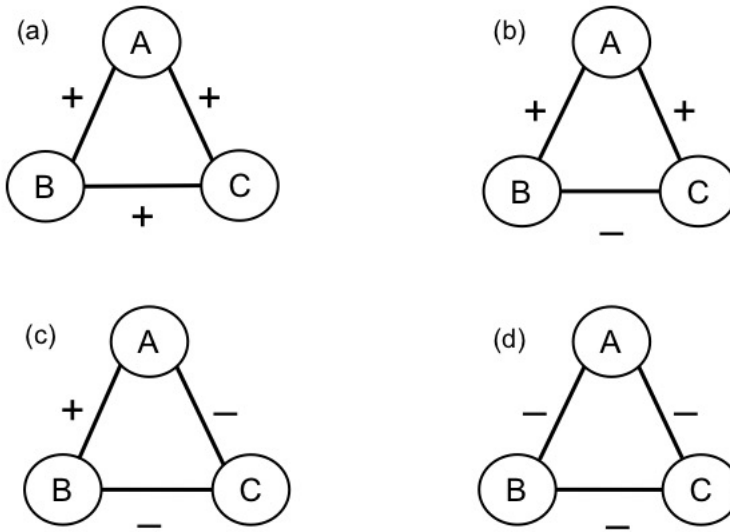


FIGURE 1. Heider's Theory of Structural Balance. This figure demonstrates the different variations of sentiment among members of a social group: (a) all members of the group are friends, forming a balanced structure (b) A has mutual friendships with B and C, but B and C are enemies, forming an unbalanced structure (c) A and B are mutual friends with a common enemy C, forming a balanced structure (d) all members are enemies, forming an unbalanced structure.

The work by Heider was generalized and formalized by Cartwright and Harary [4] through a graph-theoretic framework of larger social networks. After the construction of these foundations in social psychology, applied mathematicians were able to propose, construct, and analyze larger mathematical models of friendship. In 1977, Wayne Zachary put forth one of the earlier models of friendship [23]. In his work he explains that "the process leading to fission is viewed as an unequal flow of sentiments and information across the ties in a social network. This flow is unequal because it is uniquely constrained by the contextual range and sensitivity of each relationship in the network. The subsequent differential sharing of sentiments leads to the formation of subgroups with more internal stability than the group as a whole, and results in fission." Zachary's results were not surprising. As explained above, Heider's theory of structural balance suggests that the network will reach a socially balanced equilibrium of friendships. However, one of the more remarkable aspects of Zachary's work is the construction of a mathematical model from a raw set of data given to him by his local Karate Club, shown in Figures 2 and 3. This is an amazing feat considering that during this time applied mathematics was not as popular as today and no rigorous models of friendship existed.

In 1999 Van de Bunt et al. [21] proposed a class of actor-oriented statistical models for closed social networks in general, and friendship networks in particular. Their models are random utility models developed within a rational choice framework. Further examining social behaviors, Currarini, Jackson, and Pin develop in [3] “a model of friendship formation that sheds light on segregation patterns observed in social and economic networks.” These authors examine the properties of a “steady-state equilibrium of a matching process of friendship formation.” They use their model to investigate three empirical patterns of friendship formation: “(i) larger groups tend to form more same-type ties and fewer other-type ties than small groups, (ii) larger groups form more ties per capita, and (iii) all groups are biased towards same-type relative to demographics, with the most extreme bias coming from middle-sized groups.” The criteria behind this model’s construction echo the ideas described earlier by Oresme.

In 2010, Thomas Schwartz [15] used structural balance to develop an abstract mathematical model aimed at understanding positive and negative relationships, (e.g. friendship or enmity). He writes: “Some relational patterns (e.g. friends sharing an enemy) are balanced; others (e.g. enemies sharing a friend) are not. The model has tested well in a variety of applications, from the social psychology of small groups to the politics of international conflict. Several versions are at least implicit in the literature but had not previously been identified.” This work showcases the fact that such studies are in their infancy. In their work [7], Hassan, Salgado, and Pavón point out that “The dynamics of social relationships, such as friendship or partnership patterns, is a complex field of study. In this context, social dynamics refers to the collection of agent interactions, their conditions, and associated mechanisms, together with the emergent behaviour that they bring about.” This idea is particularly important, since we see that Oresme used among his examples mathematical ideas that today are considered by our contemporary specialists “a complex field of study.” Hassan, Salgado, and Pavón continue to explain that, “social relationships such as friendship and partner choice are ruled by the proximity principle, which states that the more similar two individuals are, the more likely they will become friends. However, proximity, similarity, and friendship are concepts with blurred edges and imprecise grades of membership.” Their study “shows how to simulate these friendship dynamics in an agent-based model that applies *fuzzy sets* theory to implement agent attributes, rules, and social relationships, explaining the process in detail.” Fuzzy logic was introduced in 1965 by Lotfi Zadeh [24] and is currently used to handle the concept of partial truth. This shows why it has taken contemporary applied mathematicians so long to approach such imprecise concepts.

In their substantive work [10], Marvel, Kleinberg, Kleinberg, and Strogatz, propose a continuous differential equation model inspired from the dynamical system $\frac{dX}{dt} = X^2$, where X is a matrix of friendliness or unfriendliness between pairs of nodes in a given network. They investigated how social networks behave in response to stress. The outcome of their analysis “implies that the initial amount of friendliness in large social networks (started from random initial conditions) determines whether they will end up in intractable conflict or global harmony.” These results show that the work within this array of problems is highly nontrivial and it involves several mathematical theories that were fully developed only in the last decades.

Aravind Srinivasan writes in [19] that “many networks of interest include explicit sentiments (positive or negative views) of the nodes (constituent entities, also known as vertices) toward each other. This is particularly the case in geopolitics, settings where networks of firms compete in the creation of standards, and situations where polarization is frequent, such as in national elections, etc.” His investigation is also rooted in the classical

work of Heider [8], which shows that the historical timeline of these mathematical models developed onward since the second half of the 20th century.

The examples of recent works selected in this section are just a few of the important contributions advancing our understanding on the territory of human relationships and social networks. More importantly, these examples illustrate how visionary Oresme's work was. By using examples from the domain of social relations for his doctrine of configurations, Oresme simultaneously attracted the attention of his readers and addressed themes of interest today.

7. CONCLUSIONS

As far as we are aware, no other parallels have been described in the scientific literature, and none of the works in applied mathematics we cite indicate Nicole Oresme among their references. Hence, no direct affiliations can be claimed between Oresme and developments in applied mathematics, in particular to sociological models. What is certain is that Oresme's works are deeply rooted in Aristotle's natural philosophy, and that this Norman-French medieval author endowed with an unusual intuition described a series of questions that are of interest today within the realm of applied mathematics. We certainly hope that our inquiries are shedding a new light on understanding the vision of this unique author, quite isolated in his century by his interesting thought.

However, if we trace a common ground between Oresme and many works written by contemporary authors in various areas of mathematical modeling, we identify an array of common problems between Aristotle's middle age heritage and questions recently investigated in applied mathematics. The present discussion is open and should be viewed as an invitation to all interested scholars to read Nicole Oresme with the attention deserved by early contributors to the field of mathematics. All with a special caveat: these are the works of an unusual visionary, as Dirk Struik was right to appreciate him as "special." Even if Oresme did not have in the 14th century the mathematical machinery to develop advanced studies compatible with our present level of rigor or content, **he did ask the right questions, and he certainly used interesting examples.** Oresme proved to have a remarkable inclination towards an applied spirit that in general is neither associated nor expected to be found in the middle ages.

The authors would like to extend their thanks to Isabel M. Serrano for our many inspiring conversations about the present topic, which is rooted into a more extensive investigation blending in both mathematics and history.

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