CREAT. MATH. INFORM.
Volume **33** (2024), No. 2,
Pages 289 - 295

Online version at https://creative-mathematics.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 286X; Online Edition: ISSN 1843 - 441X
DOI: https://doi.org/10.37193/CMI.2024.02.14

Power Domination in Generalized Mycielskian of Cycles

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ABSTRACT. Let $G = G(V, E)$ be a graph. A set $S \subseteq V$ is a power dominating set of G if S observes all the vertices in V, following two rules: domination and propagation. The cardinality of a minimum power dominating set is called the power domination number. In this paper, we compute the power domination number of generalized m-Mycielskian of a cycle, C_n. We found that it depends on the numbers n and m.

1. INTRODUCTION

A set $S \subseteq V$ is a dominating set of the graph $G(V, E)$, if each element $v \in V$ is either in S or adjacent to some element in S. The problem of finding a dominating set that has the minimum cardinality is known as dominating set problem. Many variations of domination problem were introduced and studied. The power dominating set problem is one of them. It was introduced [7] as a graphical representation of the famous observability problem of placing Phasor Measurement Units (PMUs) in an electrical network [2].

Let $G(V, E)$ represent an electric network with a vertex $v \in V$, a node and an edge $e \in E$, a transmission line. For $S \subseteq V$, the open neighbourhood of S in G, $N_G(S)$, consists of all adjacent vertices of S. $M(S)$ is the set monitored by S obtained by applying the rule of domination, initially, once and the rule of propagation iteratively, afterward as follows [4]:

- ▷ (domination rule) $M(S) \leftarrow S \cup N(S)$,
- ▷ (propagation rule) as long as there exists $v \in M(S)$ such that $N(v) \backslash M(S) = \{w\}$, set $M(S) \leftarrow M(S) \cup \{w\}$. (*i.e., propagation from v to w*)

Definition 1.1. [4] A subset $S \subseteq V$ is called a power dominating set if $M(S) = V$. A power dominating set with the minimum cardinality is called a minimum power dominating set or γ_P-set. The cardinality of a minimum power dominating set is called the power domination number, γ_P.

The problem was found to be NP-complete, even when restricted to chordal graphs or bipartite graphs [7]. However, a polynomial time algorithm was found for trees [7] and block graphs [16]. The problem was also addressed in the graph classes like grids [5], honeycomb networks [6], Knödel graphs, Hanoi graphs [14] and some other graph classes [3, 4]. The Mycielskian of a graph G, denoted by $\mu(G)$, is a triangle-free extension of G [1, 9]. The Mycielskian has a higher chromatic number than the underlying graph. The generalization of Mycielskian graphs is known as the generalized m-Mycielskian which is defined as follows:

Definition 1.2. [8] Let G be a graph with vertex set $V^0 = \{v_1^0, v_2^0, \ldots, v_n^0\}$ and edge set E^0. Given an integer $m \geq 1$, the generalized m-Mycielskian of G denoted by $\mu_m(G)$, is the graph with vertex set $V^0 \cup V^1 \cup V^2 \cup \ldots V^m \cup \{r\}$, where $V^i = \{v_j^i : v_j^0 \in V^0\}$ is the i^{th}

Received: 25.09.2023. In revised form: 10.04.2024. Accepted: 18.04.2024
2020 *Mathematics Subject Classification.* 05C69.
Key words and phrases. *Power Domination, Generalized Mycielskian, Cycles.*
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distinct copy of V^0 for $i = 1, 2, \ldots, m$ and edge set $E^0 \cup \left(\cup_{i=0}^{m-1} \left\{v_j^i v_{j'}^{i+1} : v_j^0 v_{j'}^0 \in E^0\right\}\right) \cup \left\{v_j^m r : v_j^m \in V^m\right\}$. 1-Mycielskian of a graph G is called the Mycielskian of G.

Due to the structure of Mycielskian graphs, various graph parameters were studied and compared [8]. The power domination number of Mycielskian of spiders was also computed [13] and characterized spiders in the following way.

Theorem 1.1. [13] *Let T be a spider. Then $\gamma_P(\mu(T)) = 1$ if and only if any one of the following holds.*

(i) *T is a path*
(ii) *T is a wounded spider*
(iii) *T is a single odd legged spider.*

Theorem 1.2. [13] $\gamma_P(\mu(T)) = 2$, *if and only if T is an even spider or a multiple odd legged spider which is not a wounded spider.*

The power domination number of Mycielskian of n-spiders(SP_n) was also investigated in [11]. They provided tight upper and lower bounds of γ_P of Mycielskian of n-spiders and classified n-spiders that attain both bounds, which is as follows:

Theorem 1.3. [11] *For an n-spider $SP_n,\ n > 1,\ 1 \leq \gamma_P(\mu(SP_n)) \leq n.$*

Theorem 1.4. [11] $\gamma_P(\mu(SP_n)) = 1,\ n > 1,$ *if and only if*

i. *There is no m-kid in SP_n and*
ii. *e-kids occur in pairs.*

Theorem 1.5. [11]
$\gamma_P(\mu(SP_n)) = n, n > 1,$ *if and only if*

i. *$N_m \geq n - 1$ or*
ii. *$N_m = n - 2$ and the head sequence begins (or ends) with an e-kid followed (respectively preceded) by an s-kid.*

In this paper, we compute the power domination number of m-Mycielskian of cycles, $m \geq 1$ (Figure 1).

Definition 1.3. Let C_n be a *cycle* with n vertices $c_1^0, c_2^0, \ldots, c_n^0$. If n is even we call the cycle as an *even cycle* otherwise as an *odd cycle*. c_i^j denote the twin vertex of c_i^0 in the twin set V^j.

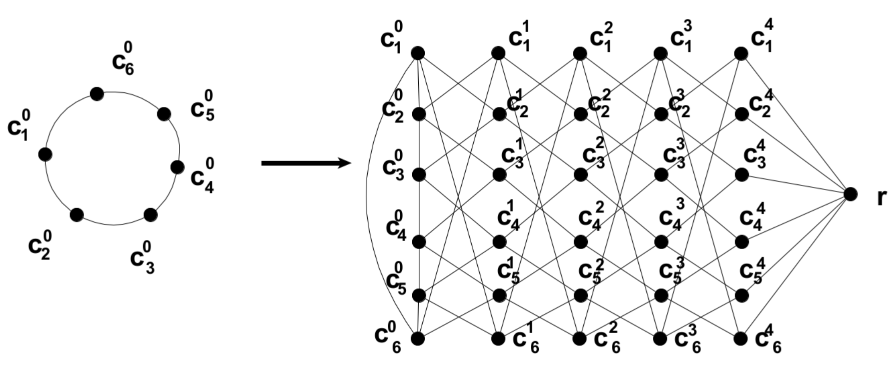

FIGURE 1. C_6 and $\mu_4(C_6)$

All graphs considered here are simple and undirected. For all the other definitions and notations refer [11, 15].

2. POWER DOMINATION IN m-MYCIELSKIAN OF CYCLES

We use the power domination subgraph relation, given by Stephen et al. in [12], to compute the power domination number of $\mu_m(C_n)$.

Theorem 2.6. [12] (Power domination-subgraph relation) *Let $H_1, H_2, \ldots, H_k$ be pair wise disjoint subgraphs of G satisfying the following conditions*

(1) $V(H_i) = V_1(H_i) \cup V_2(H_i)$ where $V_1(H_i) = \{x \in V(H_i) | x \sim y$ for some $y \in V(G) - V(H_i)\}$ and $V_2(H_i) = \{x \in V(H_i) | x \nsim y$ for all $y \in V(G) - V(H_i)\}$.

(2) $V_2(H_i) \neq \emptyset$ and for each $x \in V_1(H_i)$, there exists at least two vertices in $V_2(H_i)$ which are adjacent to x.

If $V_1(H_i)$ is observed and if γ_{P_i} is the minimum number of vertices required to observe $V(H_i)$, then $\gamma_P(G) \geq \sum_{i=1}^{k} \gamma_{P_i}$.

Lemma 2.1. *If S is a power dominating set of $\mu_m(C_n)$, then $V^i \cap N[S] \neq \emptyset$.*

Proof. If possible, suppose $V^i \cap N[S] = \emptyset$, then the only way to monitor the vertices in V^i is the propagation from V^j, $j = i+1$ or $i-1$. But each vertex in V^j is adjacent to at least two vertices in V^i. Thus no propagation is possible to V^i. Therefore S is not a PDS. □

Corollary 2.1. *For a cycle C_n, $\gamma_P(\mu_m(C_n)) \geq \lceil \frac{m+1}{3} \rceil$.*

Proof. The proof follows from the fact that there are $m+1$ twin sets $V^0, V^1, \ldots, V^m$, and for a vertex x, $N[x]$ have a non-empty intersection with at most three twin sets.

□

First, we consider the case when $m = 1$, i.e., the 1-Mycielskian of cycles.

Theorem 2.7. *For a Cycle $C_n, \gamma_P(\mu(C_n)) = \begin{cases} 1 & \text{if } n = 3 \\ 2 & \text{otherwise} \end{cases}$.*

Proof. It is easy to observe that any vertex in V^0 monitor the whole graph $\mu(C_3)$. Now, let $n > 3$. If possible, assume that $\gamma_P(\mu(C_m)) = 1$ and $S = \{x\}$, be the γ_P-set. Then by Lemma 2.1, the vertex x must be from V^0 or V^1. But, $M(x) = N[x] \subset V$ in both cases. Thus S cannot be a PDS and $\gamma_P > 1$. Now, let $S = \{c_1^0, r\}$. Then, every vertex is monitored either by domination or propagation from S. Thus $\gamma_P = 2$.

□

Now, we are computing the power domination number of m-Mycielskian of cycles, $m > 1$. Let us First consider even cycles $C_n, n > 3$. Here we are using Theorem 2.6 to obtain a lower bound to $\gamma_P(\mu_m(C_n))$. For this we are using the following types of subgraphs to form H_i (Figure 2).

i) A is the subgraph induced by the vertices
$\{c_1^0, c_2^0, \ldots, c_n^0\} \cup \{c_1^1, c_3^1, \ldots, c_{n-1}^1\}$
Then we can partition this vertex set as
$V_1(A) = \{c_1^1, c_3^1, \ldots, c_{n-1}^1\}$ and
$V_2(A) = \{c_1^0, \ldots, c_n^0\}$

ii) B_1 is the subgraph induced by the vertices
$\{c_1^m, c_3^m, \ldots, c_{n-1}^m\} \cup \{r, c_2^{m-1}, c_4^{m-1}, \ldots, c_n^{m-1}\}$.
Then we can partition this vertex set as
$V_1(B_1) = \{r, c_2^{m-1}, c_4^{m-1}, \ldots, c_n^{m-1}\}$ and
$V_2(B_1) = \{c_1^m, c_3^m, \ldots, c_{n-1}^m\}$.

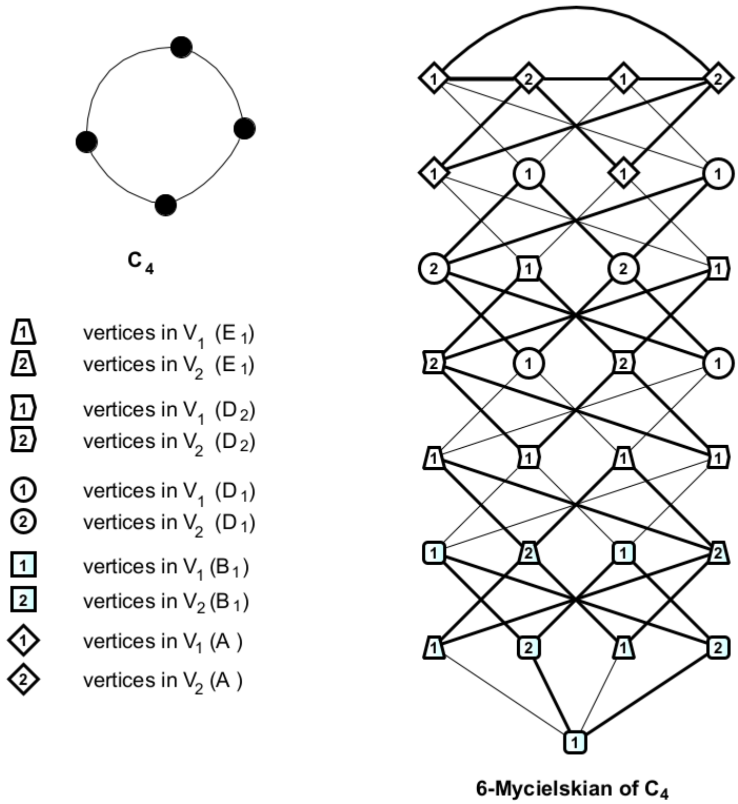

FIGURE 2. $\mu_6(C_4)$ and its partition

iii) B_2 is the subgraph induced by the vertices
$\{c_2^m, c_4^m, \ldots, c_n^m\} \cup \{r, c_1^{m-1}, c_3^{m-1}, \ldots, c_{n-1}^{m-1}\}$.
Then we can partition this vertex set as
$V_1(B_2) = \{r, c_1^{m-1}, c_1^{m-1}, \ldots, c_{n-1}\}$ and
$V_2(B_2) = \{c_2^m, c_4^m, \ldots, c_n^m\}$.

iv) D_j is the subgraph induced by the vertices
$\left\{c_2^j, c_4^j, \ldots, c_n^j\right\} \cup \left\{c_2^{j+2}, c_4^{j+2}, \ldots, c_n^{j+2}\right\} \cup \left\{c_1^{j+1}, c_3^{j+1}, \ldots, c_{n-1}^{j+1}\right\}$.
Then we can partition this vertex set as
$V_1(D_j) = \left\{c_2^j, c_4^j, \ldots, c_n^j\right\} \cup \left\{c_2^{j+2}, c_4^{j+2}, \ldots, c_n^{j+2}\right\}$ and
$V_2(D_j) = \left\{c_1^{j+1}, c_3^{j+1}, \ldots, c_{n-1}^{j+1}\right\}$.

v) E_j is the subgraph induced by the vertices
$\left\{c_1^j, c_3^j, \ldots, c_{n-1}^j\right\} \cup \left\{c_1^{j+2}, c_3^{j+2}, \ldots, c_{n-1}^{j+2}\right\} \cup \left\{c_2^{j+1}, c_4^{j+1}, \ldots, c_n^{j+1}\right\}$.
Then we can partition this vertex set as
$V_1(E_j) = \left\{c_1^j, c_3^j, \ldots, c_{n-1}^j\right\} \cup \left\{c_1^{j+2}, c_3^{j+2}, \ldots, c_{n-1}^{j+2}\right\}$ and
$V_2(E_j) = \left\{c_2^{j+1}, c_4^{j+1}, \ldots, c_n^{j+1}\right\}$.

All these subgraphs satisfy the conditions of Theorem 2.6 with $\gamma_{P_i} = 1$.

Lemma 2.2. *For an even cycle C_n, $\gamma_P(\mu_m(C_n)) \geq \begin{cases} 2\lceil \frac{m}{3} \rceil + 1, & \text{if } m \equiv 0 (mod\ 3) \\ 2\lceil \frac{m}{3} \rceil - 1, & \text{if } m \equiv 1 (mod\ 3) \\ 2\lceil \frac{m}{3} \rceil, & \text{if } m \equiv 2 (mod\ 3) \end{cases}$.*

Proof. Consider the three possible values of m.
Case 1 : $m \equiv 2(\text{mod } 3)$.
Consider the pair wise disjoint subgraphs as $D_0, E_0, D_3, E_3, \ldots, D_{m-2}, E_{m-2}$ and $\sum \gamma_{P_i} = 2\left(\frac{m+1}{3}\right) = 2\lceil \frac{m}{3} \rceil$.
Case 2 : $m \equiv 0(\text{mod } 3)$.
Case 2.1 : $m \not\equiv 0(\text{mod } 6)$.
Consider the pair wise disjoint subgraphs as $A, D_1, D_2, E_4, E_5, \ldots, D_{m-2}, B_1$ and $\sum \gamma_{P_i} = 2(\frac{m}{3}) + 1 = 2\lceil \frac{m}{3} \rceil + 1$.
Case 2.2 : $m \equiv 0(\text{mod } 6)$.
Consider the pair wise disjoint subgraphs as $A, D_1, D_2, E_4, E_5, \ldots, E_{m-2}, B_2$, then $\sum \gamma_{P_i} = 2(\frac{m}{3}) + 1 = 2\lceil \frac{m}{3} \rceil + 1$.
Case 3 : $m \equiv 1(\text{mod } 3)$.
Consider the pair wise disjoint subgraphs as $A, D_2, E_2, D_5, E_5, \ldots, D_{m-2}, E_{m-2}$, then $\sum \gamma_{P_i} = 2(\frac{m-1}{3}) + 1 = 2\lceil \frac{m}{3} \rceil - 1$.
□

Lemma 2.3. *For an even cycle C_n, $\gamma_P(\mu_m(C_n)) \leq \begin{cases} 2\lceil \frac{m}{3} \rceil + 1, & \text{if } m \equiv 0 (mod\ 3) \\ 2\lceil \frac{m}{3} \rceil, & \text{if } m \equiv 2 \text{ or } 1 (mod\ 3) \end{cases}$.*

Proof. To prove this, we provide a power dominating set, S, of required cardinality. If $m \equiv 0(\text{mod } 3)$ let $S = \left\{r, c_1^0, c_2^0, c_1^4, c_2^4, \ldots, c_1^{m-2}, c_2^{m-2}\right\}$ and if $m \equiv 1$ or $2(\text{mod } 3)$ then let $S = \left\{c_1^0, c_2^0, c_1^4, c_2^4, \ldots, c_1^{m-1}, c_2^{m-1}\right\}$. In each case, S power dominate $\mu_m(C_n)$. □

Lemma 2.4. *Let S be a power dominating set of $\mu_m(C_n), n$ is even. Then $\mid N[V^i] \cap S \mid \geq 2,\ 1 \leq i < m$.*

Proof. From Lemma 2.1, $\mid N[V^i] \cap S \mid \geq 1, 1 \leq i \leq m$. If possible let $\mid N[V^i] \cap S \mid = 1, 1 \leq i < m$ and if $N[V^i] \cap S = \{v\}$, we have two cases:
Case 1: $v \in V^i$. That is $v = c_j^i, 1 \leq j \leq n$. Then the vertex c_{j+1}^i or c_{j-1}^i is not monitored.
Case 2: $v \in V^{i+1}$ or V^{i+1}. Suppose $v \in V^{i+1}$. That is $v = c_j^{i+1}, 1 \leq j \leq n$. Then the vertex c_j^i is not monitored. Similarly, if $v \in V^{i-1}, c_j^i$ is not monitored. □

Corollary 2.2. *Let S be a power dominating set of $\mu_m(C_n), n$ is even. Then $\mid S \cap \left(V^i \cup V^{i+1} \cup V^{i+2}\right) \mid \geq 2,\ 0 \leq i < m-2$.* □

Remark 2.1. Using the same arguments in the proof of Lemma 2.4, we get the following results. Let $S \subseteq V(C_n), n$ is even.

- If $S \cap \{r\} = \emptyset$. Then, $|S \cap N[V^m]| \geq 2$.
- If $S \cap V^0 = \emptyset$. Then, $|S \cap N[V^0]| \geq 2$.

Theorem 2.8. *For an even cycle, C_n,*
$$\gamma_P(\mu_m(C_n)) = \begin{cases} 2\lceil \frac{m}{3} \rceil, & \text{if } m \equiv 1 \text{ or } 2 (mod\ 3) \\ 2\lceil \frac{m}{3} \rceil + 1, & \text{if } m \equiv 0 (mod\ 3) \end{cases}.$$

Proof. If $m \not\equiv 1(\text{mod } 3)$, the result follows from Lemmas 2.2 and 2.3. Now suppose, $m \equiv 1(\text{mod } 3)$. Consider a set $S \subseteq V(\mu_m(C_n))$ with cardinality $2\lceil \frac{m}{3} \rceil - 1$. Then we may arrange these vertices so that it satisfy the conditions in Lemmas 2.1 and 2.4, as follows:

i. Selecting exactly one vertex from each of the twin sets: $V^0, V^2, V^3, \dots, V^{3k}, V^{3k+2}, \dots, V^{m-1}$. But here, $S \cap \{r\} = \emptyset$ and $|S \cap N[V^m]| = 1$. Thus by Remark 2.1, V^m is not fully monitored.

ii. Selecting exactly one vertex from each of the twin sets: $V^1, V^2, V^4, \dots, V^{3k+1}, V^{3k+2}, \dots, V^m$. But here, $S \cap V^0 = \emptyset$ and $|S \cap N[V^0]| = 1$. Thus, by Remark 2.1, V^0 is not fully monitored.

In either way, a set with cardinality $2\lceil \frac{m}{3} \rceil - 1$ cannot be a PDS. Which gives, $\gamma_P(\mu_m(C_n)) \geq 2\lceil \frac{m}{3} \rceil$. Then by Lemma 2.3, $\gamma_P(\mu_m(C_n)) = 2\lceil \frac{m}{3} \rceil$, if $m \equiv 1 (\text{mod } 3)$. □

Now, we consider odd cycles C_n, $n \geq 3$. First let $m > 1$ and $n = 3$.

Theorem 2.9. *The power domination number of m-Mycielskian of C_3 is*

$$\gamma_P(\mu_m(C_3)) = \begin{cases} m, & \text{if } m = 2 \\ \lceil \frac{m+1}{3} \rceil, & \text{otherwise} \end{cases}.$$

Proof. For $m = 2$, suppose that $\gamma_P(\mu_2(C_3)) = 1$ and $S = \{x\}$ be a γ_P-set. Then by Lemma 2.1, the vertex x must be from V^1. But $M(x) = N[x] \subset V$. Thus S cannot be PDS and $\gamma_P > 1$. Now, let $S = \{c_1^1, r\}$. Then every vertex is monitored either by domination or propagation from S. Thus $\gamma_P = 2$.

Now, let $m > 2$. We produce a power dominating set that has cardinality $\lceil \frac{m+1}{3} \rceil$.

Case 1: $m \equiv 0 (\text{mod } 6)$, Let $S = \{c_1^0, c_1^6, \dots, c_1^m\} \cup \{c_2^3, c_2^9, \dots, c_2^{m-3}\}$.
Case 2: $m \equiv 1 (\text{mod } 6)$, Let $S = \{c_1^0, c_1^6, \dots, c_1^{m-1}\} \cup \{c_2^3, c_2^9, \dots, c_2^{m-4}\}$.
Case 3: $m \equiv 2 (\text{mod } 6)$, Let $S = \{c_1^1, c_1^7, \dots, c_1^{m-1}\} \cup \{c_2^4, c_2^{10}, \dots, c_2^{m-4}\}$.
Case 4: $m \equiv 3 (\text{mod } 6)$, Let $S = \{c_1^0, c_1^6, \dots, c_1^{m-3}\} \cup \{c_2^3, c_2^9, \dots, c_2^m\}$.
Case 5: $m \equiv 4 (\text{mod } 6)$, Let $S = \{c_1^0, c_1^6, \dots, c_1^{m-4}\} \cup \{c_2^3, c_2^9, \dots, c_2^{m-1}\}$.
Case 6: $m \equiv 5 (\text{mod } 6)$, Let $S = \{c_1^1, c_1^7, \dots, c_1^{m-4}\} \cup \{c_2^4, c_2^{10}, \dots, c_2^{m-1}\}$.

In each case S is a PDS of cardinality $\lceil \frac{m+1}{3} \rceil$. Then by Corollary 2.1, we have $\gamma_P(\mu_m(C_3)) = \lceil \frac{m+1}{3} \rceil, m \neq 2$. □

Finally, we consider odd cycles $C_n, n > 3$ and $m > 1$.

Lemma 2.5. *For the m-Mycielskian of an odd cycle, C_n, $\gamma_P(\mu_m(C_n)) \leq \lceil \frac{m+2}{3} \rceil$.*

Proof. Here, we produce a power dominating set that has cardinality $\lceil \frac{m+2}{3} \rceil$.

Case 1: $m \equiv 0 (\text{mod } 6)$, $S = \{c_1^0, c_1^6, \dots, c_1^m\} \cup \{c_2^3, c_2^9, \dots, c_2^{m-3}\}$.
Case 2: $m \equiv 1 (\text{mod } 6)$, $S = \{c_1^0, c_1^6, \dots, c_1^{m-1}\} \cup \{c_2^3, c_2^9, \dots, c_2^{m-4}\}$.
Case 3: $m \equiv 2 (\text{mod } 6)$, $S = \{c_1^1, c_1^7, \dots, c_1^{m-1}\} \cup \{c_2^4, c_2^{10}, \dots, c_2^{m-4}\} \cup \{r\}$.
Case 4: $m \equiv 3 (\text{mod } 6)$, $S = \{c_1^0, c_1^6, \dots, c_1^{m-3}\} \cup \{c_2^3, c_2^9, \dots, c_2^m\}$.
Case 5: $m \equiv 4 (\text{mod } 6)$, $S = \{c_1^0, c_1^6, \dots, c_1^{m-4}\} \cup \{c_2^3, c_2^9, \dots, c_2^{m-1}\}$.
Case 6: $m \equiv 5 (\text{mod } 6)$, $S = \{c_1^1, c_1^7, \dots, c_1^{m-4}\} \cup \{c_2^4, c_2^{10}, \dots, c_2^{m-1}\} \cup \{r\}$.

In each case S is a PDS with cardinality $\lceil \frac{m+2}{3} \rceil$. □

Theorem 2.10. *For an odd cycle, $\gamma_P(\mu_m(C_n)) = \lceil \frac{m+1}{3} \rceil, m \not\equiv 2(\text{mod} 3)$.*

Proof. By Corollary 2.1 and Lemma 2.5, $\lceil \frac{m+1}{3} \rceil \leq \gamma_P(\mu_m(C_n)) \leq \lceil \frac{m+2}{3} \rceil$. But, when $m \not\equiv 2(\text{mod } 3)$, $\lceil \frac{m+1}{3} \rceil = \lceil \frac{m+2}{3} \rceil$. Hence the result. If $m \equiv 2(\text{mod } 3)$, $\lceil \frac{m+1}{3} \rceil \leq \gamma_P(\mu_m(C_n)) \leq \lceil \frac{m+2}{3} \rceil$. □

Theorem 2.11. *For an odd cycle, $\gamma_P(\mu_m(C_n)) = \lceil \frac{m+1}{3} \rceil, m \equiv 2(\text{mod} 3)$ and $m \geq \frac{3n+1}{2}$.*

Proof. The set $S = \left\{c_1^1, c_2^4, c_3^7, \dots, c_{\frac{m+1}{3}}^{m-1}\right\}$ is a power dominating set of $\mu_m(C_n)$, where the subscripts of element of S is taken modulo n. Then the result follows from Lemma 2.1. □

3. Conclusions

The investigation of minimum placement of PMUs led to the power dominating set problem in graph theory. The problem of finding a network that has a minimum cost of installation is always relevant in electrical field. The cost of a network is proportional to the product of degree and diameter. A PMU and its installation also contribute to the network cost.

Through this study, we have found the power domination number of m-Mycielskian of cycle, C_n, on n-vertices. For even cycles, $\gamma_P(\mu_m(C_n)) = 2\lceil \frac{m}{3} \rceil + 1$ if $m \equiv 0(\text{mod}3)$ and $\gamma_P(\mu_m(C_n)) = 2\lceil \frac{m}{3} \rceil$, otherwise. It is interesting to note that, γ_P of Mycielskian of even cycles depends only on the values of m. Now, for odd cycles, if $m \equiv 2(\text{mod}3)$ and $m < \frac{3n+1}{2}$ then $\lceil \frac{m+1}{3} \rceil \leq \gamma_P(\mu_m(C_n)) \leq \lceil \frac{m+2}{3} \rceil$. For all other cases, $\gamma_P(\mu_m(C_n)) = \lceil \frac{m+1}{3} \rceil$. Also, the degree of these networks is less than $\frac{n}{m+1}$ and diameter is at most $2(m+1)$ [10]. Compared to other networks which have a less power domination number, m-Mycielskian of cycles gives a decent structure optimized with the cost-exerting factors. In addition, the structure is well-connected.

Acknowledgement. The first author acknowledges the financial support under the Junior Research Fellowship Scheme of the University Grants Commission, Government of India.

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